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## The Lemniscatory Path System PARTS 1 TO 3

An evolution of the Copernican worldview based on statements and sketches by Rudolf Steiner on the planetary movement

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## STELLARUM REVOLUTIONES Noviter Cognosces *

* You shall get to know the planets' revolutions in a new way


# THE OPPOSITES MAY FIND TOGETHER AND OUT OF THE DIVERSITIES MAY ARISE THE FINEST HARMONY. 

Heraclitus
$\sqrt[3]{3}$

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## Preliminary note

For decades, modern astrophysics was firmly convinced that in the universe observable phenomena are explained solely by the factors mass and gravitation. Meanwhile, this solution model is increasingly being called into question. Newer cosmological explanatory models demand the existence of a previously unknown "dark matter" and an equally unknown "dark energy". Both together should be about $90 \%$ of the universe's account. In the end, that means that by far the largest part of the shape-building factors in the universe is still not understood at all. Maybe different forces act together, to bring forth the shapes, movements and groupings of planets, stars and galaxies.

In such a time of searching for more "formative forces" in the universe, it may be appealing for those being on the one hand scientifically astronomical, but on the other hand also spiritually interested, to take a closer look at Rudolf Steiner's statements about planetary motion. Such a project requires a great deal of impartiality and the willingness to break away from old, cherished ideas. Up to the beginning of the 20th century, science was still firmly convinced that atoms are indivisible. Only in 1913 did Bohr succeed on the basis of the quantum laws of Planck, to describe components and the structure of the hydrogen atom. Maybe a hundred years hence, the time is ripe for the realization that movements, as we observe with the planets, are likewise not indivisible and cannot ultimately be explained atomistically, but just as well are composed of a variety of motion forces as the atoms of a variety of elementary particles, if one can still speak of particles today, because according to recent findings, these seem to be more and more an expression of forces. Would it be, out of this view, not possible that the orbit of the earth and other planets around the sun is not simply based only on gravity, but the result of the interaction of a much larger number of forces in the same way that you have a similarly complicated and multifaceted system of forces underlying the construction of an atom? Rudolf Steiner has come up with his statements and sketches for planetary motion that ventured into a whole new astronomical field of research. However, he has not left us a ready-made, self-contained teaching system about the formative forces of the planetary paths, but from different angles he described very diverse, seemingly incompatible partial views. So it is our job to put the many puzzle pieces together to form a coherent overall picture. Some parts had to be completely re-added to close the gaps in the overall picture. In the end, after many individual steps of knowledge, it has proved possible to reconcile Rudolf Steiner's statements and sketches with the Copernican system of planetary paths.

Work on this topic started in July 2008, exactly 84 years (one Uranus Cycle) after the death of Rudolf Steiner on 30 March 1925. A first script was released on 20 October 2008 and submitted to the Mathematical-Astronomical Section at the Goetheanum in Dornach. On 21 November 2009 at the invitation of the section, the findings obtained until then were presented to the Working Group "Astronomy and Spiritual Science". This talk aroused the need in the participants to have an overview of all previously obtained knowledge of other authors as well on the subject of lemniscate paths. This need was met with at the public conference "Sun, Earth and the Lemniscate" from 13 to 15 October 2010 in the Goetheanum. The session started with a presentation of the PART 1 and some of the PART 2 results described in the present work. Shortly before, in time for the meeting, PART 1 was presented in the Journal JUPITER (Vol. 5, No. 1, September 2010), published by the Mathematical-Astronomical Section at the Goetheanum. At the request of some members of the branch Kassel these results were presented on 29 January 2011 also in their Anthroposophical Center to a group of interested people under the title "The Lemniscatory Paths of the Sun and the Planets". After the work on PART 2 had been completed in the course of 2011, it again was presented to a group of interested people at the Anthroposophical Center in Kassel on 27 August 2011 under the title "The Lemniscatory Paths of Planets and the Cosmic Cross path of Sun and Earth". Shortly thereafter, the publication was in JUPITER, Vol. 6, No. 1, September 2011. The release of PART 3 in JUPITER will, according to information from Dornach, probably not be possible until 2014. In order not to wait unnecessarily long for those
interested in this subject, PARTS 1 to 3 are already provided in advance with the present document as a complete work for self-study. *

The reader is taken on an exciting journey of knowledge. It starts with the attempt of the author, first to open up the meaning of some of Rudolf Steiner's quite fundamental statements, which seem contradictory, and to explore the consequences. In doing so, the reader is constantly challenged anew to set his thinking in motion in abundance, to rise from a thinking-in-forms to a thinking-in-movements. As support, the entire work has been provided with over 250 pictures. For a long time it remains to be seen where the journey leads, whether Rudolf Steiner's statements can be reconciled with the Copernican system at all and, if so, how this can be done. It is only in PART 3 that the image is rounded off and the Lemniscatory Path System becomes recognizable as a multi-fold system of formative forces that ultimately produces the outer image of the Copernican system.

The original text of the JUPITER publications has been kept virtually unchanged. Only a few explanatory footnotes have been added. All cited statements were italicized. The somewhat bumpy and antiquated translations of the quotes from Copernicus' major work De revolutionibus in the German Thorn edition (1879) was newly translated. The illustrations have been renumbered and a list of illustrations attached.

PART 1 first introduces Rudolf Steiner's basic statements about planetary motion in our solar system. In the course of the individual steps of knowledge the reader is led further and further away from the common ideas of the Copernican system to a system of rotating lemniscatory paths.

PART 2 deepens the theme of the rotating lemniscatory paths and describes how the positions of sun and earth come to lie on straight lines in the course of the year, which together form one cosmic cross path. In this context, the phenomenon of the ecliptic is considered from a completely new perspective and the need for the so-called third Copernican Law for the Lemniscatory Path System is pointed out. Finally, the topic of the vertical and horizontal progression of lemniscatory paths is treated in detail and the apex movement of the sun included in the considerations.

Part 3 introduces as a new type of movement for the cosmic cross path of the Sun and Earth a semi-annual straight-linear forward and backward movement. On this basis, it is possible to describe how the Copernican earth's orbit is being created over five formative steps, which emerge of the interaction of twelve formative powers, which in turn can be reconciled with the twelve zodiacal forces. Ultimately, in this way, the lemniscatory paths of the planets, the cosmic cross path of sun and earth, the Copernican circular or elliptical orbit of the earth as well as its spiral path in the direction of the Solar apex, can be reconciled. In addition, the lemniscatory paths of the inner and outer planets are finally described.

Kassel, September 2012

Seven years after the first presentation of the results summarised in PART 1 at a public conference in autumn 2010 at the Goetheanum in Dornach, repeated international inquiries were made about an English translation of the present treatise. This wish can now be fulfilled. The page numbers of the English edition are the same as in the German edition. The Physicist Gopi Krishna Vijaya (Utah/USA) made a significant contribution to the translation. Therefore he deserves the express and heartfelt thanks of the author.

Kassel, December 2017

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## PART 1

Note: The suggestions of Rudolf Steiner for a more realistic view of the planetary motion seems to be incompatible with the observable orbits of the sun and planets in the sky from the Earth as well as with the validated knowledge of modern astronomy based on the Copernican-Keplerian system. In addition, Rudolf Steiner's descriptions and sketches about those, in his words, real movements are so diverse that they alone seem incompatible with each other. The concern of the following considerations, starting from Steiner's statements about lemniscatory planetary movements, is to open a path to a coherent synopsis.

### 1.0.1 Introduction

Rudolf Steiner has given us numerous suggestions for a more realistic re-examination of the planetary motion. Following the basic principle of Anthroposophy, he also has illuminated this topic from a variety of sides. But he did not leave us a ready, new model of a planetary system, but rather the task of assembling the multiplicity of his widely varying statements into a unity and reconciling it with the Copernican system prevailing today, which he considered a purely geometric image and not as reality*: "Now you have the external image, the purely geometrical external image; the other image will be added, and only from the union of the two images will later humanity gain the idea that it must have." ** (Lecture from 1 October 1916 [1])

Steiner pointed out that mankind in the course of its evolution from cultural level to cultural level has made and will continue to make different views on the planetary movement the dominant one, and that even our view today basically can only be regarded for several centuries as a part of this series. He called it therefore a "modern mythology" and stated: "A straight line goes from what the old European residents have said in their legends of gods and stars and worlds, what the Greeks, the Romans have given in their mythologies, what the Middle Ages have given in their more or less clouded mythologies, up to that mythology, which, fully suitable and totally entitled to admiration, Copernicus, Kepler and Galileo have provided. A time will come when one will talk about this modern mythology like this: There were once people who have thought it right, to place a material sun at the center of an ellipse, to let planets circle around in ellipses, to let them rotate in different ways; they have adjusted a world system like earlier times also. Today, of course, a future time will speak, all that is just legend and fairy tale. - Yes, this time will come as well, even if modern man despises the old mythologies so much and swears on his ones, and even if it seems so impossible that one can speak of a Copernican mythology." (Lecture from 12 April 1909 [2])

The path of astronomical mythology led from the planetary gods to the pure material planetary bodies. To understand their movements in the sky, at different times different systems have come to be developed. While the Ptolemaic system regarded the Earth as the center, that is, it was geocentric, the subsequent Copernican system places the sun into the center and is therefore heliocentrically constructed. Like two opposites, these two systems face each other. A similar contradiction now also seems to exist between the Copernican and the future system, which is as yet unspecified, but described in parts already by Steiner. While the Copernican system with its elliptical orbits and the sun in the middle is probably the simplest of the previous systems, it is

[^1]likely that the successor system, which we are to develop based on Steiner's statements for the future, will probably become the most complicated ever. Steiner sees the justification for this exceedingly great complexity in the fact that it portrays the real movements of the planets in space, which are so very complicated.

If one tries to get an overview of the statements of Rudolf Steiner about the real movements of the planets in space, one gets a truly amazing series of properties:

1. "We have a system of determined arranged lemniscates as the paths of the planets and also as the earth-sun path. "(Lecture from 17 January 1921 [3])
2. "The helical line continues in space. Not that the planets are moving around the sun, but these three: Mercury, Venus, Earth, follow the sun, and these three: Mars, Jupiter, Saturn go ahead." (Conference of 25 September 1919 [4]). For this the following drawing was given:


Figure 1: Rudolf Steiner's sketch of the helical line for his remarks at the conference on 25 September 1919 [4]
3. "You cannot draw into the same space the path of Venus and the path of Saturn. From this you can see ... that it is not possible to draw a solar system." (Lecture from 2 May 1920 [5]) - The "path of Venus" is here representative of the inner planets and the "path of Saturn" of the outer planets.
4. "Thus man develops geometry. But whereby does he, for example, form the idea of a triangle? ... In truth, this is an unconscious movement that he performs in the cosmos. ... The earth has quite different, artistic movements that are constantly being carried out. And much more complicated movements are carried out, such as movements, for example, lying in the lines which the geometric bodies have: the cube, the octahedron, the dodecahedron, the icosahedron, etc." (Lecture from 23 August 1919 [6])

A more self-contradictory description of the planetary movement is hard to imagine. What should a sequence of movements look like that combines all these qualities: lemniscatory paths, screw paths, which are at the same time also angular geometric paths, even emulate threedimensional bodies, and in addition not all planets run in the same space? How is the latter to be understood at all? And then Steiner says: "That it is not possible to draw a solar system." - Should one therefore refrain from all attempts to come to a more precise idea of the planetary movement from the outset? If Steiner's intention had been such, he certainly would not have given so many details, e.g. given the Lemniscate movement and even drawn numerous sketches, which nonetheless - such as Figure 1 for example - do represent a solar system.

Even if the requirement profile may be so frighteningly high, we should not be discouraged by it, but try to retrace step by step Steiner's hints, i.e. to take his information seriously, to build on it and develop it further.

The vast majority and most detailed of Steiner's information refer to Lemniscatory paths. Therefore, it makes sense to take the first step here and see if the results ultimately allow connections to the other forms of planetary movement (helix, artistic movements, polygons, platonic bodies) and whether everything can also be reconciled with the Copernican system. The following considerations are devoted to this task.

### 1.0.2 Method of calculation and Reality

For several centuries humanity has looked at the planetary motions in our solar system as circular orbits in the sense of Copernicus, with slight elliptical changes in the sense of Kepler's laws. On this basis, the planetary positions can be calculated quite well, not only for nearby times, but also for times far in the past or in the future. The fact that we are today able to use manned spacecraft to navigate and explore the moon or unmanned probes to the various planets seems to be an irrefutable confirmation of the correctness of the Copernican system.

Out in space, of course, we only ever find the planetary bodies at the calculated location. Which way they have actually taken in the space, we can neither see nor measure so far, because planets do not leave any observable trace behind them. Solely because of the accuracy of our calculation results we are firmly convinced that the Copernican-Keplerian trajectories underlying the calculations must correspond to the real trajectories of the planets in space. But is that necessarily so? - Ptolemy calculated the planetary positions based on an epicycle-system. In it, the planets move on small circular paths (epicycles), the center of which follows a large circular path. With this method results can be achieved that are in no way inferior to the Copernican methods. This fact shows that correct calculation results are by no means a proof that the planetary paths underlying the calculation must correspond to the actual paths in space. At first, they are only "assumed" trajectories, with the help of which we arrive at almost exact position determinations.

The very loopy and curvy paths of Mercury and Venus are initially a problem for the Copernican system in its original form. Modifications must be introduced to come to a solution. Thus, the orbits of Mercury and Venus must be given a particularly strong inclination to the ecliptic plane (Venus more than $3^{\circ}$ and Mercury even $7^{\circ}$ ). In addition, the orbit of Mercury must be granted an exceptionally high eccentricity ( 0.206 vs. 0.017 for Earth's orbit) and in addition, the orbit of Mercury must also perform a so-called "Perihelion precession".

More specifically, the Copernican system is thus by no means as consistent and simple as commonly thought. Therefore, the question must be allowed: Are such "concessions" to the paths of Venus and Mercury ultimately only "calculation aids" like that of Ptolemy? Or are the elliptical orbits just as "assumed" as the epicyclic paths of Ptolemy? It is conceivable that the planets run on paths that include both the mathematical laws of ellipses as well as epicycles, so that both arithmetic paths are possible without the trajectories actually being ellipses or epicycles? This would also make clear Rudolf Steiner's statement that both the elliptic system and the epicycle system are inaccurate with respect to the true planetary motions, but the latter is closer to reality: "Today, it is obvious to the human being that the sun is fixed in the middle and the planets themselves turn around in ellipses. It will not last long into the future, and one will see that Copernicus' conception of the stellar world is much more incorrect than the preceding one of Ptolemy. The Copernican-Keplerian worldview is a very comfortable worldview. But to explain what the macrocosm is, it is not the truth. " (Lecture from 18 December 1912 [7])

What do the orbits of Mercury and Venus in the sky really look like? Unfortunately, due to their proximity to the sun, both can only be observed to a very limited extent from Earth. We can only follow very small sections of their paths shortly before sunrise or shortly after sunset in the sky. We can only calculate the vast majority of their paths. Figure 2 shows such a calculated trajectory of Mercury from 15 March 2004 until 31 December 2004.


Figure 2: Calculated movement of Mercury from 15 March to 31 December 2004

The Mercury positions are shown in 5-day intervals. Mercury runs in the picture from right to left. The drawing is not quite to scale insofar as the horizontal movement of Mercury was slightly pushed together by the illustrated $270^{\circ}$ of the ecliptic, while the vertical movement of Mercury (Ecliptic latitude) was slightly pulled apart to make the loop movements more visible. However, this does not change the basic principle of the course of Mercury's path.

If one looks at this trajectory with an unbiased view, one will not be able to deny that with its rhythmic change from "loop up" and "loop down" it is reminiscent of a widely spread, so-called "advancing lemniscate" *, which moves along the ecliptic here. Could Rudolf Steiner be right in his assertion that the planets do not actually travel on elliptical orbits, but on lemniscatory paths or otherwise formed paths? And can perhaps the planetary loops of Mercury and Venus help us to confirm or refute his statements?

[^2]
### 1.0.3 Epicycle, Ellipse and Lemniscate

The fact that it is possible to arrive at almost identical results with two different calculation methods - epicyclic and elliptical - can be taken as an indication that the real trajectories of the planets may follow a superordinate principle in which the laws of both computational methods are included equally. This can be compared to the mathematical laws of a row of twelve. It contains both the regularities of the row of four ( $3 \times 4=12$ and $6 \times 4=24$ etc.) as well as the laws of the row of three $(4 \times 3=12$ and $8 \times 3=24$ etc.). In two different ways one can come to the same results of the row of twelve ( 12 and 24 ), but the sole "reality" in this example would be only the row of twelve.


Figure 3: Rudolf Steiner's sketch of the basic principle of a lemniscatory path [1]

At first glance Ellipse and Lemniscate may seem very different. But is that so? If you look at the basic principle of a lemniscatory path (Figure 3), as Rudolf Steiner gave it in the lecture from 1 October 1916 [1], you can think of these also composed of two interconnected ellipses, because the crossover in the middle of the lemniscate must by no means be always as pointed as indicated in the figure. Also two adjacent ellipses would match the basic principle of a lemniscate's form. So it would be possible to use a calculation method based on the principle of the ellipse to calculate trajectories which in reality follow the superior principle of a lemniscate.

The greatest obstacle to modern astronomy in terms of keeping a lemniscate-shaped trajectory as a possibility is likely to be the prevailing view of the forces underlying the planetary motions. Today it is assumed that all planetary motions are a result of gravitation, mass and inertia or of centripetal and centrifugal forces. The simplicity of the principle of an attractive central mass, the sun, which keeps planetary bodies wanting hurry away from it on circular orbits, is naturally impressive. Lemniscate-shaped orbits cannot be explained by this because the question of the change of direction within the lemniscate remains unanswered. But is it justifiable to regard a form of movement as impossible from the outset, just because it cannot be explained with the currently valid physical views? We should at least take a closer look at the trajectories of the planets, if they show any "oddities", that can be considered indications of lemniscatory paths. One such indication has already been mentioned above. It is the calculated trajectory of Mercury (Figure 2, page 6), which resembles an expanded "advancing lemniscate".

According to Rudolf Steiner, not only Mercury and the other planets, but also the Sun itself is running on lemniscatory paths. This implies that the Sun must change direction at certain times. The question then arises: should a change of direction in the solar movement not have an impact on the accompanying planets, especially on the adjacent planet, which is most likely to have to
participate in its movements? - Looking more closely at the Mercury path pictured above, you will find another "oddity" in it. While the middle loop in Figure 2 (page 6) looks quite elliptical or lemniscate, the loops at the beginning and end of the year are more like curves than loops. Figure 4 shows the curve that Mercury performed from March to May 2004 (enlarged). Mercury runs from right to left. Its trajectory can be better described as a "sharp point" or "acute angle with subsequent curve."


Figure 4: Calculated movement of Mercury from 15 March until 29 May 2004
This raises the question: How does the sudden change of direction in the area of the acute angle within a few days (Mercury is shown in 5-day intervals) come about? Does it not look as if it has suddenly been pulled away from its path from right to left top by a force acting on it from the outside and brought down into a curved path to the lower right? Could this acute angle in Mercury's orbit be the result of a change in the direction of the Sun on its lemniscatory path, carrying Mercury with it?

A similar phenomenon is shown by the Mercury "curve" from the end of 2004 (Figure 5) as a detail from Figure 2 (page 6). Again, it comes within a few days to a change of direction, as if Mercury would suddenly be "carried away" by the sun in another direction than its own orbit.


Figure 5: Calculated movement of Mercury from 31 October until 30 December 2004
Copernican astronomy will say that it is a matter of perspective phenomena, which is simply the result of not only Mercury, but also the earth running through the space, so that it must come to perspective distortions of the elliptical orbits. Of course, it comes to such distortions due to the constant change of location. But couldn't another factor also be involved in the phenomenon of "acute angle"? Maybe a change of direction of the sun on its lemniscatory path? The one possible and permissible explanation does not necessarily have to exclude another possible and permissible explanation.

### 1.1 The lemniscatory paths of Sun and Earth

Rudolf Steiner's claim that the Sun is moving on a lemniscatory path - which could be the cause of the "acute angles" in the course of Mercury's orbit - immediately throws up two more questions:

1. How can a lemniscatory trajectory of Sun and Earth agree with the apparent circular orbit of the sun through the zodiac?
2. How can such a pathway be reconciled with the perihelion and aphelion constellations between sun and earth (perihelion of earth on January 2 and aphelion on July 2) in the Copernican-Keplerian elliptical orbit of the earth as well as with the solstices and equinoxes?

The first question is a very fundamental one. A solution will have to be sought in the direction of a movement of the lemniscate, which would have to be of such a kind that the result would be an apparent orbit of the sun in the sky. Whether or not such a type of movement really exists and what it should ultimately look like requires a more detailed, separate consideration. It is to be postponed for the time being and is to be taken up again following the observations on the lemniscatory paths of the inner planets (see section 1.3). In addition, every movement of the lemniscate makes all the considerations so complicated that it makes sense to start with a resting lemniscate and try to answer the question about perihelion and aphelion constellations, solstices, and equinoxes on this basis. In this context, Rudolf Steiner's statement should be taken into account that the earth does not run around the sun, but: "... that we are dealing with a succession of the earth towards the sun, in a manner of hurrying ahead of the sun and a following of the earth." (Lecture from 12 January1921 [3])

### 1.1.1 Perihelion and aphelion, solstices and equinoxes in the lemniscatory path

There are some basic astronomically observable conditions for which a lemniscatory path must also provide an explanation. These are above all the different distances of the earth to the sun, the so-called perihelion constellation (near the sun) at the beginning of January or the Aphelion constellation (far from the sun) at the beginning of July, as well as the conditions of the sun's course in the sky with the sun's lowest point at the winter solstice, its highest point at the summer solstice and the equinoxes in spring and fall.

Two solutions are available for the perihelion and aphelion constellations.

### 1.1.1.1 The path length solution

On its Copernican-Keplerian elliptical orbit, the earth comes every year around 2 January nearest to the sun (Perihelion) and around 2 July furthest from the sun (Aphelion). These constellations can be reconciled with a lemniscate, assuming two differently sized lemniscate halves. One half would be the somewhat shorter perihelion or winter lemniscate half, while the other half would be the somewhat longer aphelion or summer lemniscate half (Figure 6). The consequence would be that the lemniscate would have exactly the opposite situation as the ellipse, i.e. that the perihelion and aphelion constellations come about when the earth (instead of the sun) is at the center and the sun (instead of the earth) at one of the two ends. The earth follows the course of the sun at a distance of half a semi-lemniscate.


Figure 6: Perihelion and aphelion constellations in a lemniscate with halves of different sizes

The strongly exaggerated emphasis in the figure on the elliptical shape of the two lemniscate halves as well as the greatly exaggerated difference in length between perihelion and aphelion is merely illustrative. In fact, the Keplerian ellipses are almost circular orbits. In a true-to-scale drawing, the difference between perihelion and aphelion distance would be imperceptible.

However, the movement shown in Figure 6 can only be the first step in solving the problem. If the earth would run on exactly the same path as the sun, there would be two aphelion and two perihelion constellations per year. When the sun reaches the center of the lemniscate, the earth has arrived where the sun was before. Both would have just swapped places and end up again the same distance to each other. But Rudolf Steiner points out that sun and earth each runs on its own lemniscatory path that differ in their inclination from each other and cover only in their center (lecture from 1 October 1916 [1]). The inclination angle of the Earth's path is likely to correspond to the angle between celestial equator and ecliptic, that is, the inclination of the Earth's axis of $23.5^{\circ}$ (in the Copernican system). The earth is always following the sun at a distance of a lemniscate quarter, but on its own path, which only intersects in its center with the path of the sun. Rudolf Steiner gave us the sketch of a double-lemniscate system of sun and earth (Figure 7). This shows how the sun (circle with dot - left in the picture) moves on its horizontal, brightly hatched path to the crossing point in the middle of the lemniscate, where the earth is located (circle without a dot), while it (indicated by the arrows pointing upwards) follows the course of the sun on its own darker shaded path, so to speak, or perhaps to state it better: reconstructing it on its own path.

Assuming that the lengths of the semi-lemniscates of the Earth's path are different from those of the Sun's path, the configurations of Sun's distance from the Earth and Sun's closeness to the Earth occur only once a year. Length and width of semi-lemniscates would be such that it ensures that the distance between the earth (on its path) and the sun (on its path) is always between the minimum and maximum distance of earth and sun in the Ellipse. The lemniscates would be about $2 \mathrm{AU}^{*}$ long and 1 AU wide. Thus, the basic conditions of perihelion / aphelion constellation would also be fulfilled in a Lemniscatory Path System.

[^3]

Figure 7: Rudolf Steiner's sketch of a double-lemniscate system of Sun and Earth [1]

### 1.1.1.2 The speed solution

Another solution is that the sun's greatest distance and greatest closeness are not an integral part of the path length of the sun, but is caused by a rhythmically changing and decreasing velocity of the earth on its path. This frees the positions at the endpoints of the lemniscate-halves to serve as solstice positions. The situation indicated by Rudolf Steiner in his sketch (Figure 7) would then correspond to the time of the winter solstice. In Figure 8 it is shown on the top left, how the sun passes through the lowest point under the inclined plane of the earth's path. The Earth's path lies in the same plane, which corresponds to the celestial equator in Copernican terms.


Figure 8: Solstices and Equinoxes in the double-lemniscate system of Sun and Earth

The earth's axis is perpendicular to the Earth's path. In order to always maintain a distance of 1 AU between the sun and the earth, a width of 0.977 AU must be assumed for the elliptical lemniscate halves of their paths. So they are almost circular. The picture at the lower left shows the situation at the summer solstice. The sun has reached its highest point above Earth's path plane.

Under Earth's conditions, one would explain the maximum speed of the earth a few days after the winter solstice in the perihelion by the fact that the sun accelerates with the "down-swing" up to a certain value, so that the slowing down forces of the ascent are noticed somewhat delayed. Likewise, the minimum speed after the summer solstice in the aphelion could be explained by the
fact that the sun loses its speed when climbing up until slightly above the maximum level, whereby also here it takes a few days before the accelerating forces of descent recur. Of course, in space we have no gravity of the earth as the cause of the decline and the slowdown of the ascent. However, the question arises as to whether this change of speed may not be based on a general developmental principle, according to which an upward development always requires more power and proceeds more slowly than a downward trend. Likewise, it would correspond to a very natural rhythm of tension and relaxation, of activity and passivity, which of course may seem absurd in the context of a mechanistic world view, but certainly makes sense if one allows the existence of rhythms of life also in the cosmos. The basic principle of all life processes is precisely the rhythm of blossoming and withering, expansion and contraction, of inhalation and exhalation, and rhythm is equally the rationale of everything planetary. Rhythms of life and planetary rhythms are closely related to one another. The latter even seem to be the causes of the former. Just think of the monthly rhythm of the woman, which corresponds to the 28 -day lunar rhythm, or the rhythm of vegetation on the earth, which follows the annual rhythm of the sun's course.

In the right half of Figure 8 the situation of the equinoxes is shown. The sun crosses the Earth path plane, which is Copernican the crossing of the celestial equator.

### 1.1.2 The advancing lemniscate and the path-shaping forces

Now the actual movement of sun and earth is not to be imagined as shown before, so that both of them would constantly move around on their eternally same path like in a lying eight, but at the same time an upward movement is connected with this movement in the sense of an "advancing lemniscates" as described by Steiner in the so-called "Third Natural Science Course" (Lecture from 12 January 1921 [3], Fig. 6):


Figure 9: Rudolf Steiner's sketch of an advancing lemniscate [3]

Rudolf Steiner adds to this form of lemniscate that "its axis itself becomes a lemniscate again" (Lecture from 2 May 1920 [5]). This movement of the axes should initially be disregarded, as it is easier to observe the movements of the sun and the earth on a resting, upwardly advancing lemniscatory path.

At this point we will briefly discuss the path-shaping forces. Lemniscatory paths, whether they are self-contained or progressive, obviously cannot be explained by the force model of the Copernican system. This raises the question: where do the forces for such trajectories come from? Rudolf Steiner gave us the following short explanation: "... if we take this outermost planet of our
solar system, Saturn, then we have to imagine it ... as the leader of our planetary system in world space. It pulls our planetary system in the world space. It is the body of the outermost force that guides us around in the lemniscate in the world space. It drives and pulls at the same time. So it is the force of the outermost periphery. If it worked alone, we would only move in the lemniscate. But now in our planetary system there are these other forces, which represent a more intimate mediation to the spiritual world, which we find in Mercury and in Venus. These forces continually lift the path. So that when we look at this path from above, we get this lemniscate (the previous curve); but when we look at them from the side, we get lines that are constantly rising, advancing."(Lecture from 2 May 1920 [5])

The conjecture is therefore obvious that all planets are involved in any way in these paths and that a whole series of forces acts, which allows not only lemniscatory paths, but also the others mentioned in the "introduction", like helices, polygons etc. which are still to be discovered. Unfortunately, Rudolf Steiner did not leave us with more information on this question of forces.

Based on Figures 6 to 9, the representation of a common, advancing double-lemniscatory path of the Sun and Earth can be developed (see Figure 10). The form of an advancing lemniscate given by Rudolf Steiner is used for the earth's path. The sun rises and falls rhythmically against this path. It does not take into account the inclination of the lemniscate axis in relation to the north celestial pole, the lemniscatory axis movement and the exact distances between the sun path and the earth path. In order to determine whether the numerous loop variants of the inner planets are caused by a lemniscatory inherent motion of the sun, i.e. whether they are in a way indicators of the sun's movement, the working model shown in Figure 10 is sufficient.

22 December
Earth-Sun Path


Figure 10: Advancing double-lemniscatory path of Sun and Earth
In all of the following considerations, the "speed solution" of the perihelion-aphelion problem is assumed. Sun and earth are therefore in the constellation of the winter solstice, i.e. the earth is the center of the two lemniscates or where the two path lines cross each other (earth path blue, Sun path orange) and the sun is at the end of the lemniscate half. In order to be able to start the winter solstice in the lower right-hand loop (lemniscate half), the positions of Figure 7 (Page 11) or Figure 8 upper left part of the picture (Page 11) must be rotated by $180^{\circ}$. So the sun is running forward instead of backward, and the earth is running backward instead of forward. The zodiac position of the sun on 22 December of a year is $0^{\circ}$ Capricorn. The vernal point lies approximately in the middle in the direction of the observer. The sun is about to approach it via the signs Aquarius and Pisces.

As far as Mercury and Venus are concerned, we know that they maintain relatively constant distances to the Sun (within perihelion aphelion fluctuations). The closer the planets are to the Sun, the more their movement must be influenced by the lemniscatory movement of the sun, i.e. their path must be bound in some way to the path of the sun. Rudolf Steiner also gave us a sketch of this with the words: "An inner planet has a path that makes a loop, the center of which is the earth-sun path itself." (see Figure 11 from the lecture of 17 January 1921 [3])


Figure 11: Rudolf Steiner‘s sketch of the lemniscatory paths of Mercury and Venus [3]
By connecting the Mercury and Venus paths with the intersections in their Lemniscate center to the Earth-Sun path, Steiner shows that these two planets must somehow join in the lemniscatory movement of the Sun. He further says: "The sighting line (v) is here. We get here the loop (s) and these two branches seemingly run to infinity ( $u$ ). " In the second half of this sentence, one wonders how well this phrase is to be understood: "seemingly run to infinity". Apparently, Steiner wants to suggest that the real paths look quite different than they are drawn here and that he has made things much simpler in order to be able to represent the linkage of the paths at all. So he adds: "... the matter itself is so extraordinarily complicated that one can only get to the schematic conceptions."

Although the latter quote by Steiner may seem a bit discouraging, we have a whole series of guidelines and indications on the basis of which we can venture, taking into account the circumstances of the Copernican system, to establish a link between the two systems, in order to arrive at more concrete conceptions of the trajectories and perhaps even to explanations of such strange phenomena as the "acute angle" in the course of the Mercury path. On this basis, now the considerations on the lemniscatory paths of the inner planets are to take place.

### 1.2 The lemniscatory paths of the inner planets

### 1.2.1 Procedure for determining the planetary lemniscates

In the task of reconciling the motion sequences of the Lemniscatory Path System described by Steiner with those of the Copernican system, one can in principle follow two ways: one of which is to try to integrate the lemniscatory paths into the Copernican system. But this results in a fundamental problem. The Copernican system is based on the idea of a sun resting in the center of the system ("resting" at least within the frame of reference Sun - Planets). Now, however, the lemniscatory path system, as Rudolf Steiner describes it, is based on a lemniscaticorily moving sun with effects on the paths of the planets it carries along. Therefore, it makes sense to do exactly the reverse, i.e. to use the lemniscatory path system according to Rudolf Steiner's specifications as a basis and to integrate the elliptical orbits of the Copernican system into it.

The following considerations are based on the idea that the sun draws the planets along with it on its lemniscatory path. But because the planets in the sense of the Copernican system cannot break out of their elliptical orbits, one must assume that each position of a planet on its elliptical orbit always coincides with a position still to be determined on a further probably lemniscate-like path that belongs to it, whose exact form would also still have to be determined. As the sun, surrounded by the Copernican orbit of a planet, wanders along its lemniscatory path, in addition to the elliptical orbit of the planet a further trajectory is created.


Figure 12: Basic lemniscatory system of Earth / Sun with the elliptical orbit of Mercury
If we use the "advancing double-lemniscatory path of Sun and Earth" shown in Figure 10 (page 13) and add the Copernican elliptical orbit of Mercury, the result is Figure 12 *.

The position of the observer should be imagined in such way, that one is standing in front of the path system, which runs from bottom to top. At the same time, the gaze enters the system

[^4]obliquely from above. The lower part of each loop (lemniscate quarter) is therefore to be thought of as going forward and the upper part as going backwards. The small arrow above the earth points obliquely backwards and the small arrow under the sun points obliquely forward. The earth is initially running on its blue path away from the viewer, the sun on its orange path towards the viewer.

The lemniscatory paths of sun and earth intersect in their common center. In Figure 12, the earth is where it can be assumed for the time of the winter solstice in the lemniscate. The crossing point or lemniscate center is alternately traversed by the earth and the sun. So we get a system that is heliocentric at one time and geocentric at another and could therefore be called a "heliogeocentric system". In the light of our considerations, the sun initially moves forward on its lemniscatory path (downwards in the drawing) and then up to the next crossing point just above the earth. If one connects the crossing points (lemniscate midpoints) one obtains a directional indication, which would correspond to the movement of the sun towards the apex in the Lemniscatory Path System (dashed violet line).

### 1.2.2 The Lemniscate of Mercury

Based on Figure 12 (page 15), Mercury's path can now be graphically determined in the Lemniscate System. By adding the Copernican orbit of Mercury around the Sun (dashed green line - Figure 13) it is possible to position Mercury within the frame of reference Earth-Sun. If you refer back to the Copernican data of the year 2003, the sun stood like every year in $0^{\circ}$ Capricorn at the winter solstice. At the same time Mercury stood in $11^{\circ}$ Capricorn and approached its lower conjunction, i.e. it stood between the sun and the earth. Its angular distance from the sun was $11^{\circ}$ on 22 December 2003 from Earth's point of view. It can be a maximum of $28^{\circ}$ (maximum elongation).

In monthly steps, the other positions of Mercury in the lemniscate system are now determined graphically. These are roughly drawn by eye, so they make no claim to astronomical-mathematical or geometrical exactness. All determined positions of Mercury are taken over with the date to the illustration of the following month, in order to be able to follow the course of Mercury's movement. The current position of Mercury is marked with a red date. Also, the small circle representing the current position of Mercury is on the elliptical orbit and is also slightly darker colored than the small circles indicating the previous positions of Mercury.

You can see how the sun on its lemniscatory path simply leads Mercury's elliptical orbit with it. The arrow, which passes through the sun from the earth, points in the direction of the zodiac position of the sun. It has already been pointed out above that the solar movement on a lemniscatory path seen from the earth does not simultaneously allow a circular motion through the zodiac. In order to do this, the lemniscates path itself would have to move, rotate or oscillate as well, as is to be explored subsequently to the considerations of the lemniscatory paths of the inner planets. To include this circumstance would, however, complicate the considerations intended here even more. Therefore, a resting lemniscate is to be assumed, in which inevitably the zodiacal positions are very widely distributed in space or jump around (of course, this also applies to the vernal equinox), and later, it should be explained how that can be reconciled with an apparent circular path of the sun through the zodiac.


Figure 13: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 22 December 2003


Figure 14: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 21 January 2004. The position of Mercury on 22 December 2003 was retained.


Figure 15: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 19 February 2004. The positions of Mercury on 22 December 2003 and 19 February 2004 were retained at monthly intervals.

At this point, it is already evident the individual positions of Mercury form a very unique trajectory, as it must be assumed when the sun moves on a lemniscatory path, although the
positions of Mercury on its elliptical orbit still maintain their validity, while the sun takes the laws of the Copernican system with it.


Figure 16: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 20 March 2004. The positions of Mercury from 22 December 2003 to 20 March 2004 were retained at monthly intervals.


Figure 17: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 20 April 2004. The positions of Mercury from 22 December 2003 to 20 April 2004 were retained at monthly intervals.

As an interim result, Figure 18 shows for the first time an excerpt from the lemniscate-like course of the path of Mercury, as it can be assumed for the first half of 2004 in connection with the base lemniscate system of the sun and the earth. It is interesting that here too the "acute angle" can be found again as a pathway element (Mercury's position on 20 April 2004).

In Figure 21 you can see the now extended path section of Mercury in the year 2004. To the acute angle of 20 April 2004 a loop was added in the middle of the year (from 21 June via 22 July to 23 August 04).


Figure 18: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 21 May 2004. The positions of Mercury from 22 December 2003 to 21 May 2004 were retained at monthly intervals. The path of Mercury was graphically determined.


Figure 19: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 21 June 2004. The positions of Mercury from 22 December 2003 to 21 June 2004 were retained at monthly intervals.


Figure 20: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 22 July 2004. The positions of Mercury from 22 December 2003 to 22 July 2004 were retained at monthly intervals.


Figure 21: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 23 August 2004. The positions of Mercury from 22 December 2003 to 23 August 2004 were retained at monthly intervals. The path of Mercury was graphically determined.


Figure 22: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 23 September 2004. The positions of Mercury from 22 December 2003 to 23 September 2004 were retained at monthly intervals.


Figure 23: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 23 October 2004. The positions of Mercury from 22 December 2003 to 23 October 2004 were retained at monthly intervals.


Figure 24: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 22 November 2004. The positions of Mercury from 22 December 2003 to 22 November 2004 were retained at monthly intervals.


Figure 25: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 22 December 2004. The positions of Mercury from 22 December 2003 to 22 December 2004 were retained at monthly intervals. The path of Mercury was graphically determined over a period of one year.


Figure 26: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 20 January 2005. The positions of Mercury from 22 December 2003 to 20 January 2005 were retained at monthly intervals.


Figure 27: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 19 February 2005. Die Merkurpositionen vom 22.12.2003 bis 19.02.2005 wurden im Abstand von jeweils einem Monat festgehalten.


Figure 28: Base lemniscate system of Earth / Sun with the elliptical orbit of Mercury on 21 March 2005. The positions of Mercury from 22 December 2003 to 21 March 2005 were retained at monthly intervals. The path of Mercury was graphically determined.


Figure 29: The lemniscate of Mercury from December 2003 to March 2005
Figures 25, 28 and 29 show that towards the end of the year 2004 the path of Mercury again forms an "acute angle".

On the 21st of March 2005 the sun reaches the upper end of the Earth-Sun-Lemniscate on which our considerations are based. As a result, Figure 29 shows the trajectory of the lemniscates of Mercury over one and a half years, from December 2003 to March 2005. It can be seen that Mercury's trajectory is not a simple lemniscate, but rather a form that has both basic elements of a lemniscatory path: loops and change of direction. Ultimately, the shape is formed by the sun, which simply pulls Mercury along with its lemniscatory run.

Next, the question arises: Can the graphically determined course of Mercury's lemniscate be reconciled with the astronomical observation or the calculable form of Mercury's path? Or are there at least some similarities? Due to its proximity to the Sun, Mercury is unfortunately only observable from Earth for very limited periods of time, namely when it is far enough away from the sun so that it is no longer outshone by sunlight. These are the times near to its greatest western and greatest eastern "elongation", where it is visible for a short time in the morning or evening sky. The fact that the loops and curves at Mercury occur near to its maximum elongations already excludes the possibility of justifying them with the usual explanation for planetary loops, according to which the loops are to be the result of a planet's overtaking the Earth. What is certainly true for the outer planets*, is more difficult to apply to the inner planets. One may certainly take the point of view that every loop or curve of Mercury results from an entire orbit of Mercury around the sun, which can also be reconciled with an elliptical orbit, if one only makes sufficient concessions (especially strong inclination to the ecliptic plane, extraordinary high orbital eccentricity and also a "rotation of the perihelion", i.e. apsidal precession). But here, assuming impartiality, it is obvious to ask: do other possible causes for the formation of Mercury's loops and curves arise from a lemniscatory path of Mercury, as it results from the above considerations?

Here, only a direct comparison between the graphically determined trajectory of Mercury and its Copernican calculable trajectory can help. That this trajectory looks very "lemniscatory" has already been shown in Figure 2 (page 6). Mercury runs from right to left through the zodiac (ecliptic longitude).

In Figure 30 you can see the lemniscate of Mercury on the left and a path section from Figure 2 (page 6) on the right. On the lemniscate the comparable path section is highlighted by red dates. Both drawings clearly show an "acute angle". One could, of course, argue that this must of course also be found on the lemniscate, because it is included in the calculated and conferred zodiac positions. However, it would remain unnoticed that Mercury's zodiac positions are moved to completely different places by the strong positional changes of the sun in the course of its lemniscatory path, which is also the reason for the already mentioned need for further consideration of a possible compatibility with the apparent circular path of the sun through the zodiac. (See below.) In a resting lemniscate, the vernal equinox moves strongly back and forth along with the zodiac positions indicated. So it is quite astonishing to find a similar path element on the lemniscatory path of Mercury within a comparable period of time as on the elliptical orbit. The shape differences of the two "acute angles" can be explained by the different positions from which the path is viewed. In the case of the lemniscate, the observer stands outside the earth-sun path and looks diagonally from above onto or into it. In the calculated Mercury movement, one has to think of the beholder to be on the earth and from there looking out to Mercury, with the earth moving on. This must necessarily result in perspective changes and also slight temporal displacements of the angle's form.

[^5]

Figure 30: Comparison of Mercury's path from March to May 2004
Nevertheless, the "match" found could just be a coincidence. If Mercury's lemniscate-shaped trajectory is partly responsible for its loops and curves, then this must also apply to the other conspicuous pathway elements of 2004. It is therefore to be examined whether the Copernican loop of the middle of the year can also be found on the lemniscatory path. Figure 31 provides information on this. In fact, there is also a loop formation in the middle of the year.


Figure 31: Comparison of Mercury's path from July to September 2004
Similarly, the "acute angle" calculated on a Copernican basis at the end of the year is presented in the comparable section of the lemniscatory path of Mercury again. See Figure 32.


Figure 32: Comparison of Mercury's path from October to December 2004

Ultimately, in the period of 2004 and the first quarter of 2005, all path elements that result from the Copernican calculation are also reflected in the course of the path of Mercury line, as they must result for the assumed Lemniscatory Path System, although the observer's point of view is quite different from that in the Copernican system.

Also on the question of how "acute angles" and "loops" come at all, the determined figures provide information:

- Figure 16 (page 18) shows how, at the end of March 2004, Mercury begins to run in the opposite direction to the sun, as it pulls the planet over or up to the next half of the lemniscate. The reason for the acute angle of Figure 30 (page 24) is apparently the retrograde action of Mercury towards the Sun with simultaneous swiveling of the sun in the direction of movement of the other lemniscate half.
- By the middle of 2004, Mercury is again regressive to the sun. This retains its running direction within the lemniscate half (Figure 19, page 19). The consequence for Mercury's path is a looping (Figure 21, page 20, and Figure 31, page 24).
- On 23 October 2004, Mercury and Sun again run in opposite directions (Figure 23, page 19). The sun is still in the process of changing lemniscate halves, which is associated with a change of direction of its path. This creates an acute angle again. See Figure 32 (pag 24).

The actual cause for the emergence of acute angles and loops in Mercury's path obviously lies in the occasionally occurring opposite movements of Sun and Mercury in connection with a change of direction of the sun caused by the lemniscatory path. In addition to the two already known explanations of the planetary loops, on the one hand based on the Copernican-Keplerian elliptical orbit system and on the other hand based on the Ptolemaic epicyclic system, now a third explanation is added, which results from the laws of the Lemniscate System.

The actual cause for the emergence of sharp angles and loops in the Mercury orbit is apparently the occasionally occurring opposite motions of the sun and Mercury in connection with a change of direction of the sun caused by the lemniscate orbit. In addition to the two already known explanations of the planetary loops, on the one hand on the basis of the CopernicanKeplerian elliptical orbit system and on the other hand on the basis of the Ptolemaic epicycle system, there is now a third explanation possibility, which arises from the laws of the lemniscate orbit system.

If one compares the looping of an outer planet with that of an inner planet, it turns out that in both cases a movement forward and backward is responsible for it. However, there is one major difference: In the outer planets, the overtaking and retreating planet is the Earth and the looping planet just one of the outer planets. The observable looping is a pseudo-movement, a kind of perspective projection of the Earth's movement. Copernicus himself has already recognized this connection. For the inner planets, however, the overtaking and retreating planet itself is also the loop-forming planet. Therefore, the rule for the outer planets cannot be transferred directly to the inner planets. Although the number of loop formations also corresponds here to the number of cycles around the sun (from the point of view of the Copernican system), i.e. the number of advances and regressions, the extreme variability of the loops, which are often only acute angles, sharp points or even S-curves, which usually makes the term "loop" seem inaccurate, cannot be explained by this alone. However, in order to be able to grasp them in the Copernican-Keplerian way, one transferred the motion sequences of the sun to Mercury or the Mercury's orbit. This required the introduction of exceptions such as a particularly strong orbit inclination compared to the ecliptic plane, an extremely high orbital eccentricity and additionally a rotation of the orbit itself in the form of the so-called perihelion rotation or apsidal precession. By adding further computational elements one approaches the observable form just like Ptolemy with his epicycles.

Also in the Lemniscatory Path System the loop formation is created by a kind of overtaking. But: The run-ahead and backward migration does not occur towards the Earth, but towards the sun. The shape variants acute angles or sharp points and S-curves (for the latter can be found an example in the following consideration of the lemniscates of Venus) arise from the fact that the sun draws Mercury along with it in its changes of direction, in the course of its lemniscatory path, and thus has a form-shaping effect on the "loops". Mercury's loopy outswings are an indicator of this self-movement of the sun. Of course, Copernicus could not arrive at such an explanation, since he assumed that the sun was resting. There is no need to introduce any special rules for Mercury. Loops and acute angles result naturally from the given movement sequences.

A further difference between the loop formation of the outer and inner planets - actually a contradiction - lies in the fact that with the inner planets, the "planet" that is being overtaken (here the sun, which in the lemniscate system is also a "wandering one", or a "planet"), has an active formative effect on the path of the overtaking planet, while with the outer planets the planet being overtaken is passive in this respect. (This can be said at least in the narrower sense. In a broader sense, according to Rudolf Steiner, Mars carries along the entire inner planetary system along its lemniscatory path, thus having a path-forming effect on all inner planets, including Earth and Sun. Whether this has an effect on its loop shape in any way, can only be checked in the context of separate considerations to the lemniscatory paths of the outer planets.)

On the spatial extent of Mercury's lemniscate the following statements can be made. See Figure 33 (page 27): The common Earth-Sun lemniscate has a longitudinal diameter of about 2 AU and a transverse diameter of about 1 AU . Mercury can travel on all sides at most about its distance from the sun beyond the Earth-Sun lemniscate. This corresponds to a maximum longitudinal diameter of Mercury's lemniscate of $2 \mathrm{AU}+2 \times 0.4 \mathrm{AU}=2.8 \mathrm{AU}$ and a maximum transverse diameter of Mercury's lemniscate of $1 \mathrm{AU}+2 \times 0.4 \mathrm{AE}=1.8 \mathrm{AU}$. Since the nearest planet Venus draws its course at a greater distance from the sun, Venus also runs further away from the common Earth-Sun lemniscate through space. Thus, in the Lemniscate Path System Mercury is the planet closest to the sun and the earth, because Sun and Earth together form the center of the system, at least as far as the inner planets are concerned.

Thus, the statement of Rudolf Steiner "An inner planet has a path that makes a loop whose center is the Earth-Sun path itself" (Lecture from 17 January 1921 [3]) can be interpreted in two respects. First, he points out that the lemniscatory path of an inner planet is bound to the course of the sun on the Earth-Sun path. On the other hand, Steiner may also have meant by this statement that the Earth-Sun path in its entirety forms the center within the lemniscatory path of Mercury, as it can be seen from Figure 33, and also results from the maximum conceivable longitudinal and transverse diameters of Mercury's lemniscate. The first interpretation provides yet another insight. If one looks at the way in which the Mercury's path is bound to the Earth-Sun path, it turns out that Mercury's path movement is epicyclic. It moves in a constant circular direction, whereby the center of the circle moves along the lemniscatory Earth-Sun path. The way of thinking of the Ptolemaic epicyclic system could not only be applied from a mathematical point of view, but it would also correspond to the actual trajectory of Mercury in the lemniscate system. Rudolf Steiner's statement, quoted at the beginning in the section "Calculation method and Reality" is confirmed here: "It will not take long into the future, and one will see that the view of Copernicus from the world of stars is much more incorrect than that of Ptolemy. The Copernican-Keplerian worldview is a very comfortable worldview. But to explain what the macrocosm is, it is not the truth. " (Lecture from 18 December 1912 [7])


Figure 33: Longitudinal diameter of the lemniscate of Mercury

### 1.2.3 The Lemniscate of Venus

For the graphical determination of the course of the lemniscatory path of the planet Venus, the coordinates from December 2003 to March 2005 were used as in the considerations about the planet Mercury.

The elliptical orbit of Venus around the Sun has been inserted into the base lemniscate system of Earth / Sun, whereby its suspected own motion is initially disregarded here. The considerations begin with the position of earth and sun at the time of the winter solstice on 22 December 2003. The sun stood in $0^{\circ}$ Capricorn. This direction is indicated by the arrow that goes from the earth through the sun. In all figures it indicates the direction of view to the apparent position of the sun in the zodiac. Venus stood in $1^{\circ}$ Aquarius the same day. Its positioning on the elliptical orbit is based on its angular distance from the sun, which is indicated on each figure together with the coordinates at the top right. The maximum angular distance between Sun and Venus is $47^{\circ}$. The illustrations do not claim astronomical-mathematical or geometrical exactness. The individual positions of Earth, Sun and Venus are through rough positioning by eye and should only reflect the principal sequence of movements. The individual, graphically determined positions of Venus are always taken over with the associated date on the subsequent images, so that Venus' movement in the lemniscate system can be traced step by step. Each current position of Venus is provided with a red date and also recognizable by the fact that the small circle representing Venus is located on the elliptical orbit and is slightly darker colored than the small circles marking the previous positions of Venus.


Figure 34: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 22 December 2003


Figure 35: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 21 January 2004. The position of Venus on 22 December 2003 was retained.


Figure 36: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 19 February 2004. The positions of Venus on 22 December 2003 and 19 February 2004 were retained in monthly intervals.


Figure 37: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 20 March 2004. The positions of Venus from 22 December 2003 to 20 March 2004 were retained in monthly intervals.


Figure 38: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 20 April 2004. The positions of Venus from 22 December 2003 to 20 April 2004 were retained in monthly intervals.


Figure 39: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 21 May 2004. The positions of Venus from 22 December 2003 to 21 May 2004 were retained in monthly intervals.


Figure 40: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 21 June 2004. The positions of Venus from 22 December 2003 to 21 June 2004 were retained in monthly intervals. The path of Venus was graphically determined.

Figure 40 shows the trajectory of Venus as it results for the first half of 2004. This shows a wide S-curve-like movement towards the middle of the year. If one looks at how this arises, it turns out that the retrograde motion of Venus from the Sun extends over the entire period of the semilemniscate change of the sun. This is imprinted on the trajectory of Venus, which travels epicyclically along the solar lemniscate. Presumably it is due to the larger diameter of Venus' path, that here there is no acute angle as in the case of Mercury, but instead a kind of S-curve is created.


Figure 41: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 22 July 2004. The positions of Venus from 22 December 2003 to 22 July 2004 were retained in monthly intervals.


Figure 42: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 23 August 2004. The positions of Venus from 22 December 2003 to 23 August 2004 were retained in monthly intervals.


Figure 43: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 23 September 2004. The positions of Venus from 22 December 2003 to 23 September 2004 were retained in monthly intervals.


Figure 44: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 23 October 2004. The positions of Venus from 22 December 2003 to 23 October 2004 were retained in monthly intervals.


Figure 45: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 22 November 2004. The positions of Venus from 22 December 2003 to 22 November 2004 were retained in monthly intervals.


Figure 46: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 22 December 2004. The positions of Venus from 22 December 2003 to 22 December 2004 were retained in monthly intervals. The path of Venus was graphically determined.


Figure 47: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 20 January 2005. The positions of Venus from 22 December 2003 to 20 January 2005 were retained in monthly intervals.


Figure 48: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 19 February 2005. The positions of Venus from 22 December 2003 to 19 February 2005 were retained in monthly intervals.


Figure 49: Base lemniscate system of Earth / Sun with the elliptical orbit of Venus on 21 March 2005. The positions of Venus from 22 December 2003 to 21 March 2005 were retained in monthly intervals. The path of Venus was graphically determined.

The overview of the year 2004 in Figure 46 of the trajectory of Venus shows that no further conspicuous path outswings occur after the "S-curve" at the middle of the year. The loop from 21 May 2004 to 23 August 2004 is basically still part of the S-curve and so extensive - almost as large as a semi-lemniscate loop of the sun - that seen from Earth (or Copernican calculated) it probably should not be observed as a conspicuous part of the path.

Figure 49 shows the determined lemniscate of Venus over a period of almost one and a half years. Venus' change of direction from clockwise to counterclockwise in February to April 2004 means the transition of Venus into its second half of the lemniscate, whereby the cause for this and the resulting path-forming is the movement of the sun. It is evident how extensive the semilemniscates of Venus' path are and that they lose their distinction as loops in the distance and certainly cannot be distinguished from the imaginary circular orbit from the earth's perspective. Perhaps Steiner meant this circumstance when he said about the lemniscate loops of the inner planets that the loops he marked with "u" "seemingly run to infinity". See Figure 11 (page 14).

Solving the trajectory of Venus' lemniscate out of the Earth/Sun base system, the form of the lemniscate of Venus depicted in Figure 50 is obtained from December 2003 to March 2005.


Figure 50: The lemniscate of Venus from December 2003 to March 2005
What then results in comparison with the Copernican calculable trajectory of Venus, as it would show for the view from Earth, if Venus would be observable throughout the year? Figure 51 shows this trajectory. The ecliptic latitude is overemphasized here in relation to the ecliptic longitude in order show the movement of Venus towards the middle of the year more clearly.


Ecliptic Longitude
Figure 51: Calculated movement of Venus from 22 December 2003 to 21 March 2005
The conspicuous orbital elements (Venus runs in the figure from right to left) are a large S-curve of Venus' path shortly before the middle of the year and a long-drawn outgoing curve arc. Now it should be checked whether the calculated S-curve falls within the same period as the one graphically determined. Figure 52 confirms the match.


Figure 52: Comparison of Venus' path from March to August 2004

Finally, we will briefly discuss the extent of the lemniscate of Venus in relation to the EarthSun path in its "center". Figure 53 shows that the lemniscatory path of Venus can extend beyond the Earth-Sun path on both sides by a maximum of one radius of its elliptical orbit and thus have a maximum longitudinal diameter of $2 \mathrm{AU}+2 \times$ approx. $0.75 \mathrm{AU}=3.5 \mathrm{AU}$. Accordingly, the maximum transverse diameter of the lemniscate of Venus is $1 \mathrm{AU}+$ approx. $2 \times 0.75 \mathrm{AU}=$ 2.5 AU .


Figure 53: Longitudinal diameter of the lemniscate of Venus
Because Venus' lemniscate, in relation to the Earth-Sun path in its "center", runs much further out in space than the path of Mercury, the closest planet to Sun and Earth is Mercury, whereas Venus is the second closest similar to the planetary order in the Ptolemaic system. In addition, the term "inner planets" loses its justification, as the trajectories of both Mercury and Venus run halfway out of the Earth-Sun path. Therefore, they can at best be described as "earth-sun-pathbound planets" or "center-oriented planets" in contrast to the "outer planets", which, according to Steiner, lead the entire "inner system" through space and orientate themselves to the periphery, thus being "peripheral-oriented".

### 1.3 Lemniscatory or circular path of the sun?

In the considerations on the lemniscatory paths orbits of the inner planets, the observer's gaze always went in the direction of the central common earth-sun-lemniscate. The situation of sun and planets in the lemniscate can be compared to that of the train crew in a moving train, travelling regularly from front to back and from back to front again, which has is its own, describable laws, regardless of which movements the train itself makes. Whether the train is travelling forwards, backwards or in a circle is irrelevant for the "inner-train" movement sequences. In this way, one can also observe the movements of the sun and the planets in the lemniscate without taking into account the motion relationship between the sun and the starry sky. The question as to whether and
how the lemniscatory movement of the sun can be reconciled with its apparent circular path in the sky could remain open at first. The question as to whether and how the lemniscatory movement of the sun can be reconciled with its apparent circular path in the sky could remain open at first. This fundamental problem will now be examined in more detail.

At first glance, motion on a lemniscatory path seems totally incompatible with that on a circular path. Due to the change of direction within the lemniscate, the sun and the earth tear themselves away, so to speak, from the circular path. It has already been pointed out that a solution can only be found in the fact that the lemniscatory path itself carries out a kind of rotary movement which compensates the change of direction to such an extent that the apparent orbit of the sun in the sky results. It would also be conceivable that the circular path of sun at sky could be result of several additional motions. For this purpose, the following solution options are available:

1. Rotary motion of the Earth-Sun lemniscate
2. Rotary motion of the starry sky
3. Rotary motion of the entire system of all lemniscatory paths
4. Composite motion consisting of a rotary motion of the Earth-Sun lemniscate and a rotary motion of the starry sky or the entire system of all lemniscatory paths

These solutions options will be considered in more detail below.

### 1.3.1 Rotary motion of the Earth-Sun lemniscate

Rudolf Steiner has left us a sketch in one of his notebooks (notice sheet 121), which can be interpreted as an indication of a rotation of the Earth-Sun lemniscate (Figure 54). The sketch is similar to Figure 11 (page 14), but additionally contains a form which apparently is to represent half of a lemniscate. The way it is drawn can be interpreted as a left turn of the lemniscate, with the associated Venus or Mercury path being carried along.


Figure 54: Rudolf Steiner’s sketch from Notebook 121
Such a left turn of the lemniscate can be used as a basis for further considerations. This raises the question as to what exactly this movement should look like. A continuous rotation always in the same direction cannot cancel the change of direction within the lemniscate. Since within a full lemniscatory cycle the sun runs "left around" for half a year and "right around" for half a year, the
lemniscate should also have to turn half a year "left around" and half a year "right around" to compensate for this change of motion. This would be consistent with the basic principle of all planetary phenomena: rhythmic movement.

If one examines in which month the Earth-Sun lemniscate would have to turn in which direction by how many degrees in order to allow an apparent circular path of the Sun in the sky, a first graphical-sketchy rollover yields the following result:

|  | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rotation | $\mathbf{- 9 0}^{\circ}$ | $\mathbf{- 9 0}^{\circ}$ | $\mathbf{- 9 0}^{\circ}$ | $\mathbf{- 7 0}^{\circ}$ | $\mathbf{- 2 0}^{\circ}$ | $\mathbf{0}^{\circ}$ | $\mathbf{+ 2 0}^{\circ}$ | $\mathbf{+ 7 0}^{\circ}$ | $\mathbf{0}^{\circ}$ | $\mathbf{0}^{\circ}$ | $\mathbf{- 2 0}^{\circ}$ | $\mathbf{- 7 0}^{\circ}$ |

The Earth-Sun lemniscate would have to rotate in the first three months at a constant speed by $-90^{\circ}$ (to the left). From April onwards, this movement would have to slow down to $-70^{\circ}$ until it would only be $-20^{\circ}$ in May. In June, the lemniscate would have to rest for some time. From July onwards, a gradually increasing clockwise rotation would have to take place, from $+20^{\circ}$ to $+70^{\circ}$. Between the $+70^{\circ}$ in August and the $0^{\circ}$ in September, a decreasing motion must be assumed, so that the $+70^{\circ}$ may spread over two months and thus halve to $2 \times\left(+35^{\circ}\right)$. Then the lemniscate would have to stand still and turn left again in November/December. Such a movement would actually cancel the change of direction in the lemniscate, so that the sun can make an apparent circular path in the sky. The only problem is that the durations of the left turn and the right turn are very unequal. Also, the lemniscate would be resting for a month in June, but two months in September and October. Such a movement appears to be too uneven and arrhythmic. It can hardly be considered an acceptable solution to the problem. If, nevertheless, there is a rotation of the lemniscate, an additional kind of movement should have to occur which would cancel the unevenness of the above solution.

### 1.3.2 Rotary motion of the starry sky

As an additional motion - beyond a rotation of the lemniscate - a rotation of the starry sky could also be considered for a completely unbiased approach. However, this may seem completely absurd to the contemporary scientific view of space. It is now taken for granted that we look out into an infinite space in which the earth is only a tiny dot under billions and billions of other tiny dots like a grain of sand on the seashore. Therefore, the idea of a rotating space with its gigantic proportions seems downright absurd. And because today we feel so sure about our "enlightened" conception of the universe, we don't dare to question it any more. The term "outer space" virtually cements the idea that the whole world is only "space" and that the same spatial conditions prevail in the "depths of space" as in the near-earth space. Should not we at least consider the possibility that space could also be divided into qualitatively different areas, each with its own conditions? A strange phenomenon in this context seems to be the realization of modern astronomy that you will soon be unable to cope with spatial dimensions, the further you get from the earth. For the description of large distances in extraterrestrial space, the propagation speed of light has been taken as a basis for calculations and spatial measures have been replaced by temporal measures such as the light year, the light hour or light minute. It is also assumed today that the view to the starry sky does not provide us with a picture of the cosmic present, but more and more parts of the past mix into the picture the farther the view penetrates into the depths of the universe. It is likely that a large number of the more distant stars and star systems no longer even exist in the sense of modern astronomical conception. Does not the question have to arise: do we really only look into space when we observe the starry sky or do we not look much further into time as well? More
precisely: into the past? Is what we see there perhaps a kind of "spatial image" that "reflects" the past? A kind of "cosmic picture gallery of eternity"? Does this even form a unity in the end? And maybe this also has its own movement - a turning movement?

The fact that modern astronomy has to change from spatial measures to temporal measures to be able to describe the phenomena in space shows, that science has long since reached a point where the path must lead beyond purely spatial thinking. One just does not dare to take this step. There is still no basis for understanding that the phenomenon of time, which seems so mysterious, wants to blend into the phenomenon of space. Therefore, it makes sense to first of all make some very basic thoughts on space and time, as they might arise from the anthroposophical point of view.

### 1.3.2.1 Space, time, eternity

If one wants to include in a few words the vastly varied statements of anthroposophy as Rudolf Steiner has given us, one could describe them as the "Trinitarian teaching of modern times". Like a red thread, the principle of the Trinity passes through all its cycles of lectures, no matter what subject area they are dedicated to in particular. The anthroposophical conception of the world as a whole is also trinitarian. It is divided into a spiritual world, a soul world and a bodily or physical world. Each of these three worlds has certain basic properties. Thus, for example, everything bodily can be described with spatial terms. All psychological experience takes place in the temporal, starting with the first memory through contemporary experience to the most daring ideas of a distant future. But everything spiritual is eternal. The ideas that underlie our terms can be expressed in various ways in the psychological and the bodily, and yet in their existence they are independent of their respective forms of expression.


Figure 55: Space, time, eternity - the three worlds
These three worlds are in a certain relationship to each other. While the spiritual world (the eternal) forms a kind of opposition to the bodily or physical world (the transient), the soul-world acts as a link between the two. Figure 55 shows how the soul-world protrudes into the other two worlds and thus also carries its basic property of time into them.

Each of these three worlds is additionally reflected in the other two. This makes each world in turn threefold. The physical world is divided into a lowest level in which the spatial is expressed particularly strongly. Above it lies a plane into which the temporal is clearly interfering. This is the plane of the planets, including sun and moon. Earlier cultures knew that. They read the time division in years and months on the course of the sun and the moon and named the days of the week according to the planets (planetary gods), which has been preserved to this day. The upper part of the physical world is a plane in which (nearly) eternal, immutable things show up, which is why the objects in the starry sky are called fixed stars. In this context it is interesting that even science on the edge of the universe is looking for the beginning of time. There, time springs from timelessness, i.e. of eternity. See Figure 56. The universe itself tells us that everything started in the periphery, everything is formed out of the periphery. Modern research just cannot imagine this. It attempts to condense the periphery with the origin of time into an imaginary primordial center out of which everything has exploded out with a big bang. Here, development processes of the earth are projected out into the cosmos in an inadmissible way, such as the sprouting of a plant out of a seed or the growth of an organism out of an egg cell - although neither of these occur in a chaotic and explosive way, but are arranged in a wise way. Likewise, the spatial interrelation of the earth and the near-earth space is projected unchanged into the starry sky. Both ideas are not in line with the perceived phenomenon that a look into space corresponds to a glance into the past.


Figure 56: The reflection of the three worlds in the physical world
On the earth, only what is there at the same time can stand next to each other spatially, Simultaneity is a property of space, that has not yet been scientifically described. Only if the time difference $=0$ between two objects, any spatial difference can be described. It makes no sense to want to describe the spatial distance between an object of today and an object of three weeks ago. They are not spatially related. For them, the space difference $=0$ and the time difference $=3$ weeks. These are fixed laws, as they apply to the earth. In the starry sky, on the other hand, we find objects from very different times side by side. This clearly shows that we must not simply project the laws of space, as we experience it on earth, unaltered into space. Instead, we have to ask ourselves: is not what we are seeing there more of an "image" than a spatial object? Temporal standing side by side can very well be combined to a picture. An image in turn can form a unity and such a unit can also show a rotation. Thus, if one takes the risk of imagining that the lawfulness of space fades out with increasing
distance in the universe and is gradually replaced or at least permeated by the laws of time and finally even by eternity - i.e. that spatial-object-like changes to temporal-pictorial - then also the idea of a rotation of the starry sky becomes possible.

Nonetheless, this idea does not only create discomfort from a modern astronomical point of view, but also from an anthroposophical point of view. The latter regards the starry sky as a sensual image of the region of eternity. A complete rest can thus be brought into harmony with this rather than a movement, albeit regular. The principle of motion, which is a temporal event, is the basic principle of everything planetary, including the sun and the moon. We will therefore have to look for another solution.

### 1.3.3 Rotary motion of the entire planetary system (system rotation)

It would also be conceivable that the entire planetary system would form a unit, a type of "organism" within which a multiplicity of motion sequences take place. Both natural science and anthroposophy assume that at the very beginning of the formation of the planetary system there was a uniform substantiality (natural science: gas - anthroposophy: heat), which was set in rotation at a certain stage of development. It is quite possible that this original rotary movement was maintained even after the original substance differentiated into different materials and planets. In this way, there would still be a system rotation that would be added to all other path movements of the planets. In this way, the rotation would be taken off from the starry sky and shifted to the planetary region. However, the direction of rotation would have to reverse (see below). This solution appears more probable. However, it is also more complicated than the previous one, because here the Earth-Sun lemniscate - like all other planetary paths also - performs two types of motion at the same time: the own path motion plus the system rotation.

### 1.3.4 Pivoting motion of the Earth-Sun lemniscate and rotary motion of the starry sky

Although a rotary motion of the entire planetary system (system rotation) is more likely, nevertheless, for purely practical reasons, initially a rotary motion of the starry sky should be assumed, because this variant offers the possibility of considering two overlapping sequences of motion initially completely separate from each other and thus to be able to examine the laws of each one undisturbed from the other.

What could the two types of movement look like? If you experiment with different directions of rotation and speeds of rotation, you can actually find a solution - maybe there are others? - which does justice to both the apparent circular path of the sun in the sky as well as a lemniscatory path of the sun and earth.

Assuming that the starry sky performs a smooth, even rotation of - $30^{\circ}$ per month (left turn), the result is as follows:

|  | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Earth-Sun <br> Lemniscate | -60 | -60 | -60 | -40 | 0 | +40 | +60 | +60 | +60 | +40 | 0 | -40 |
| Starry sky | -30 | -30 | -30 | -30 | -30 | -30 | -30 | -30 | -30 | -30 | -30 | -30 |

The table shows an evenly rhythmic movement sequence for the Sun lemniscate. Its rotation slows in April by the same number of degrees, with which it increases again after the temporary halt in May then in June in the opposite direction. The same happens from October to December. This corresponds to a smooth, rhythmic pivoting back and forth. In addition, there is a steadily calm rotation of the starry sky by $-30^{\circ}$ per month. This solution can be converted to the following graphical representations.

In Figure 57, the Earth lemniscate and the Sun lemniscate have been combined to a common Earth-Sun lemniscate. The observer looks down on this vertically from above. The upper lemniscate half is shown darker (thick black circle) than the lower half. This is to express that the upper lemniscate half of the sun's path is during the dark half of the year and the lower lemniscate half during the bright half of the year. The zodiac is drawn around the lemniscate. The center of the zodiac coincides with the position of the earth because from there we look out into space at the apparent circular path of the sun in the sky. The chosen time is the winter solstice on 22 December of a year. The sun is seen from the earth in $0^{\circ}$ Capricorn. The blue dashed lines in the winter lemniscate half divide the circular path into equal sections. Each segment corresponds to the distance travelled by the sun and earth in one month.


Figure 57: Position of the Earth-Sun lemniscate in relation to the zodiac on 22 December with a rotating starry sky

A solution of the problem would thus come about if:

- Sun and earth first run clockwise (yellow and blue arrow) on the common lemniscatory path. The direction will be reversed later in the second half of the lemniscate.
- At the same time, the lemniscate rotates counterclockwise for half a year and then clockwise, each at $260^{\circ}\left(3 \times 60^{\circ}+2 \times 40^{\circ}\right)$. The illustrations start on 22 December with a left turn by $-60^{\circ}$ per month.
- And at the same time also the zodiac or the starry sky turns by $-30^{\circ}$ per month also to the left, counterclockwise. This movement is retained unchanged.

Again, the trinitarian principle seems to be effective, by the cooperation of three independent movements. Since with such a view everything is in motion, one has to choose a "fixed point" in order to be able to comprehend the individual movements at all. This means that one of the factors involved must maintain its position without disregarding the prescribed laws. Here it is advisable to fix the lemniscate midpoint in the center of all illustrations. The lemniscate then pivots around this center, whereby the earth and the sun move on the lemniscate.


Figure 58: Position of the Earth-Sun lemniscate in relation to the zodiac on 21 January with a rotating starry sky

Figure 58 shows the situation on 21 January. Sun and Earth have covered their monthly distance on the lemniscate, the lemniscate itself has turned $60^{\circ}$ to the left and the starry sky has also turned $30^{\circ}$ to the left. Since the position of the sun in the zodiac is always to be thought of as seen from the earth, the center of the zodiac must move together with the Earth out of the center of the picture and along the lemniscatory path. The sun is now in $0^{\circ}$ Aquarius. The direction in which the yellow arrow points, which passes through the sun from the earth, corresponds to a kind of cosmic east. Just as on earth in the East every two hours another constellation ascends, in the same way here every month another constellation takes the place. The pivoting motion of the lemniscate and the proper movements of the sun and earth thus lead to a "standstill" of the sun in the sky. In order for it to pass through the zodiac, another motion must be added, that does not come from the sun and the earth, and possibly not from the lemniscate. This motion is carried out here first of all by the starry sky or the zodiac itself. What else can be responsible for this motion will be looked at in more detail below.

Another left-hand turn of the lemniscate by $-60^{\circ}$ gives the situation as it can be seen for the 19 February in Figure 59. The sun is now in $0^{\circ}$ Pisces. The center of the zodiac has moved on with the earth.


Figure 59: Position of the Earth-Sun lemniscate in relation to the zodiac on 19 February with a rotating starry sky

In Figure 60, the sun changes to the summer lemniscate half. This is the time of the vernal equinox, when the sun is at the center of the lemniscate.


Figure 60: Position of the Earth-Sun lemniscate in relation to the zodiac on 20 March with a rotating starry sky


Figure 61: Position of the Earth-Sun lemniscate in relation to the zodiac on 20 April with a rotating starry sky


Figure 62: Position of the Earth-Sun lemniscate in relation to the zodiac on 21 May with a rotating starry sky


Figure 63: Position of the Earth-Sun lemniscate in relation to the zodiac on 21 June with a rotating starry sky


Figure 64: Position of the Earth-Sun lemniscate in relation to the zodiac on 22 July with a rotating starry sky


Figure 65: Position of the Earth-Sun lemniscate in relation to the zodiac on 23 August with a rotating starry sky


Figure 66: Position of the Earth-Sun lemniscate in relation to the zodiac on 23 September with a rotating starry sky


Figure 67: Position of the Earth-Sun lemniscate in relation to the zodiac on 23 October with a rotating starry sky


Figure 68: Position of the Earth-Sun lemniscate in relation to the zodiac on 22 November with a rotating starry sky


Figure 69: Position of the Earth-Sun lemniscate in relation to the zodiac on 22 December. with a rotating starry sky

This would be a solution that allows the sun and the earth to run on a lemniscatory path and yet, looked from the earth, the sun makes a circular path in the sky.

If we now transfer - similar to the procedure for determining the lemniscates of Mercury and Venus - all sun and earth positions of Figures 57 to 69 on a separate sheet, we obtain a quite surprising result, which is recorded in Figure 70. The sun and the earth seem to perform their movements in a completely different space area. This underlines Rudolf Steiner's statement that the earth does not run around the sun at all. Only twice a year do they pass through the same place, the center of the lemniscate. On 22 December the earth passes through the place where the sun was on 23 September, and on 21 June it passes through the place where the sun was on 20 March.


Figure 70: Sun and Earth movement in a pivoting lemniscate

At first glance, this sequence of movements seems to create a circle with a lemniscate in the interior both for the path of the sun and for the path of the earth. However, this is an "apparent lemniscates" composed of two S-curves, as shown by the resolution of the Sun's total annual motion in its four seasonal sections (Figure 71).


Figure 71: Sun movement - cutouts
In winter and summer the sun runs a semicircular path, in spring and fall an S-curve. Both are elements that we also find in the lemniscate, and yet they are combined here in a different way. Figures 70 and 71 , however, are purely planar. In the sense of a progressive lemniscate, they must be raised to the three-dimensional. If one assumes a continuous upward movement of the sun, the figure 72 results. The space cylinder on the left is rotated by $90^{\circ}$ in relation to the two "ground plans" (right) - the 22 December is shifted to the lower left - because in this perspective the trajectory of the sun in perspective can be better represented. The different colors and types of lines help to harmonize the path sections on the right with those on the left in the picture. The transverse diameter of the space cylinder is approx. 1 AU.


Figure 72: Helical movement of the sun in a progressive pivoting lemniscate
The sun, in its course on the lemniscate pivoting back and forth, and now also advancing, makes a helical movement, which alternately contains semicircular and S-curvy sections. The upward movement of the sun is in direction of the Apex. For the first time, it is understandable that
planets moving on lemniscatory trajectories can simultaneously follow a trajectory that can be called helical path in simplified terms. In fact, it is a helical path with S-curvy sections. Since, according to Steiner, the outer planets also run on lemniscatory paths, which may be assumed to perform a rhythmic pivoting motion as well, Mars, Jupiter and Saturn will also have a helical path similar to that sketched by Steiner himself (Figure 1, page 4). This sketch should not be considered too rigidly. It can only reflect in principle, because, of course, the planets arranged on the helix must be allowed to perform their own movements, otherwise, for example, a Saturn-Jupiter opposition could never come about, in which the earth must pass between Saturn and Jupiter. So perhaps the helix sketched by Steiner represents a simplified S-curve helix, as it results for an advancing pivoting lemniscate of Saturn. Thus, the apparent incompatibility of lemniscatory paths and the sketched helical path can be resolved.

For the earth itself, there is still a slightly different pattern of movement, as the Earth's orbit is inclined to the sun's orbit by $23.5^{\circ}$. This could be disregarded in the above perspective with the view vertically from above. When looking from the side, this aspect must be included. Figure 73 shows the trajectory of the earth in a non-advancing pivoting lemniscate. Due to the pivoting motion, the two lemniscate halves of the Earth path are pushed together and turned.

Looking at the trajectory of the earth, as it results in a non-advancing pivoting lemniscate, one is reminded of Rudolf Steiner's statement: "The earth has still completely different, artistic movements, which are constantly being carried out there." (Lecture from 23 August 1919 [6])


Figure 73: Trajectory of the Earth in a non-advancing pivoting lemniscate (artistic movements)

### 1.3.5 Pivoting motion of the Earth-Sun lemniscate with simultaneous rotary motion in the context of the system rotation

So far, the pivoting motion of the Earth-Sun lemniscate was independent of a simultaneous rotary motion in the context of considering an assumed rotation of the entire planetary system. For purely practical reasons - in order to be able to study the pivoting motion of the Earth-Sun lemniscate undisturbed - this was "transferred" to the starry sky. But, if this is to rest now and yet the sun's path in the sky is to run on a circular path through the zodiac constellations, the Earth-Sun lemniscate must additionally perform the general system rotation.

In the Copernican system, a rotation of the entire system is conceivable in principle, but it is extremely unlikely. The outer planets would have to complete a full orbit within a year. In the lemniscatory system also, a common system rotation of all planets is unthinkable, because the outer planets lead the entire inner planetary system on a helical path through space. Consequently, the center of rotation could only be located in the center of Saturn's path and not in the center of the Earth-Sun lemniscate. However, a solution to the problem is still possible if one considers the entire inner planetary system as one unit, which performs a common system rotation by $360^{\circ}$ per year. The inner and outer planetary systems would each form a unit with different to inverse laws, with the sun acting as a link between the two. Above, the contrast in the loop formation of the inner planets compared to the loop formation of the outer planets has already been pointed out, as well as that the inner planets are center-oriented while the outer planets are peripheral oriented. The annual rotation of the inner planetary system is transmitted to Earth in the Copernican system. This results in the conception of an annual orbit of the earth around the sun.

In order to be able to visualize this thought and the associated processes, "fixed points" must again be selected. In this case, it is advisable to maintain the fixation of the middle point of the lemniscate in the middle of the picture and also to fix the starry sky. However, the "resting" of the starry sky is only relative, as the earth moves on the lemniscate and carries with it the "center" of the zodiac. Nevertheless, the point $0^{\circ}$ Capricorn from now on should always stay vertically above the earth and also the spatial directions of the other signs of the zodiac should be maintained. For this purpose, the pivoting lemniscate must turn by $+30^{\circ}$ per month (clockwise). The home position is shown in Figure 74.


Figure 74: Position of the Earth-Sun lemniscate with resting zodiac on 22 December.


Figure 75: Position of the Earth-Sun lemniscate with resting zodiac on 21 January


Figure 76: Position of the Earth-Sun lemniscate with resting zodiac on 19 February


Figure 77: Position of the Earth-Sun lemniscate with resting zodiac on 20 March


Figure 78: Position of the Earth-Sun lemniscate with resting zodiac on 20 April


Figure 79: Position of the Earth-Sun lemniscate with resting zodiac on 21 May


Figure 80: Position of the Earth-Sun lemniscate with resting zodiac on 21 June


Figure 81: Position of the Earth-Sun lemniscate with resting zodiac on 22 July


Figure 82: Position of the Earth-Sun lemniscate with resting zodiac on 23 August


Figure 83: Position of the Earth-Sun lemniscate with resting zodiac on 23 September


Figure 84: Position of the Earth-Sun lemniscate with resting zodiac on 23 October


Figure 85: Position of the Earth-Sun lemniscate with resting zodiac on 22 November


Figure 86: Position of the Earth-Sun lemniscate with resting zodiac on 22 December

If the Earth-Sun lemniscates, in addition to its rhythmic pivoting motion, performs a continuous $+30^{\circ}$ rotary motion per month, the Sun, viewed from Earth (!), travels evenly through the zodiac, as shown in Figures 74 to 86.

After the movements of sun and earth in the given reference frame of a rhythmic pivoting motion and a continuous rotary motion of the Earth-Sun lemniscates, resp. the entire inner planetary system, taking place at the same time, the individual positions can be transferred to a separate sheet. Again, the result is a completely surprising picture, which is shown in Figure 87. Instead of curves you now find straight-linear path sections.


Figure 87: Straight-linear path sections of Sun and Earth
If you follow the earth's movement from 22 December until 20 March, it results in a straight line, which runs horizontally from the middle of the picture to the left. The following movements are also such that they result in straight lines instead of curves. This phenomenon, the fact that flowing spatial movements of various kinds (rotary, pivoting and lemniscatory motion) can ultimately result in straight lines, is reminiscent of a statement by Rudolf Steiner, which seems so far from all his other descriptions of the planetary movements: "The earth not only has the movement that it has in the Copernican world view: it has quite different, artistic movements, which are constantly being carried out there. And much more complicated movements are carried out, such movements, for example, that lie in the lines, which have the geometrical bodies: the cube, the octahedron, the dodecahedron, the icosahedron and so on." (Lecture from 23 August 1919 [6]).

Figure 88 shows the straight-linear path sections of Sun and Earth separated from each other for clarity. Do not the words attributed to Plato suggest themselves here: "God geometrizes?"

Next, it should be checked whether these lines are consistent with the platonic bodies. The above statement of Steiner is probably not to be interpreted in such a way that Sun and Earth reproduce the platonic bodies in full three-dimensional form. They both would have to run up and down too often. Basically it can only be meant that the inclination angles of the straight-linear path sections correspond to the inclination angles of the edge lines of the platonic bodies, so that sun and earth run along some of these lines. This in turn requires a closer examination of the up and down movements of Sun and Earth in the advancing lemniscate. The considerations on this and on the lemniscatory axis movement of the Earth-Sun lemniscate are to be published in one of the following JUPITER editions. ${ }^{*}$ See PART 2 of this text starting from page 65.

This took place in September 2011.


Figure 88: Separate representation of the straight-linear path sections of Sun and Earth
If one looks at all the results obtained so far, it becomes apparent how much the descriptions of the trajectories of the sun and the earth depend on the underlying reference system. Ultimately, however, the answer to the introducing either / or question given in the chapter heading of 1.3 "Lemniscatory or circular path of the Sun?" can be given as: A lemniscatory path of the sun by no means excludes a circular path in the sky, if one permits further movement sequences. On the contrary, there are even more pathway variants of a completely different kind, which all, taken alone, seem to be totally incompatible with each other at first glance, and yet, as the above explanations show, can exist very well next to each other. Thus, numerous statements by Rudolf Steiner find their confirmation.

The consequences that ultimately result for the path of the earth in space, will now be summed up again.

### 1.4 The path of the earth in outer space

All trajectories of the earth described so far raise the question of whether they can be verified in any way using stellar parallax measurements. It must be said that neither the lemniscatory movement of the sun and the earth, nor the helical-S-curvy, nor the straight-linear motion found, is likely to reflect the exact actual spatial course of motion in the universe. Because, according to Rudolf Steiner, Mars takes the Earth-Sun lemniscate with it on the journey on its own lemniscatory path. Figure 89 shows a sketch of Rudolf Steiner, which he commented with the following words: "... an outer planet takes up the Earth-Sun path in its loop"- However, this does not remain immobile within the Martian loop, but moves forward with Mars. For, as he continues: "But now the lemniscate advances, so it presses itself through through this lemniscate, which represents the outer planets." * (Lecture from 17 January 1921 [3])

A further form of movement of the Earth-Sun lemniscate in outer space, independent of the rhythmic pivoting motion and the continuous rotary motion, must be assumed just because Mars, when it changes into its second lemniscate half, must maintain its distance to the sun in the sense

[^6]of the Copernican system. This inevitably means that the entire Earth-Sun path is carried along by Mars. The sun shows here a double nature. While it behaves in a path-forming manner for the inner planets and the earth, i. e. actively, it is apparently passive in relation to the outer planets. It can be carried along and thus gets additional forms of movement imprinted on its own trajectory. Of course, all this also affects the movement of the earth in space. Since the lemniscate of Mars probably also carries out a rhythmic pivoting motion, this has a further influence on the trajectory of the sun and earth in space. If you also assume apart from that - following Rudolf Steiner's instructions - that Mars is taken along with the Earth-Sun path by Jupiter on its lemniscatory course, that Jupiter's lemniscate may also have a pivoting motion, and then Jupiter is carried along by Saturn on its lemniscate, which may also perform a pivoting motion, then only, with taking into account all these influences, one would come to a trajectory of the earth, for which it would make sense to try to verify it by means of star parallax measurements.


Figure 89: Rudolf Steiner’ sketch on the Earth-Sun lemniscate with the lemniscatory paths of the inner and outer planets

The above considerations make it clear that every change of the reference system, every inclusion of a further lawfulness in the planetary system, leads to a course of motion which is quite unique in itself but nonetheless valid within the corresponding frame of reference. In the following, the movements of the earth found so far within the different frames of reference will be summarized again:

- The Copernican system is based on the idea of a sun resting in the center of the system. The movement of the earth presents itself as a circular or elliptical orbit around the sun.
- Modern astronomy assumes that the sun does not stand still in space, but moves straightlinearly in the direction of the solar apex. The movement of the earth changes thereby to a helical motion, when its two-dimensional circular or elliptical orbit is pulled in the threedimensional by the upward movement of the sun.
- According to Rudolf Steiner, the sun does not move straight-linearly but in the sense of an advancing lemniscate. The direction of advancing of the lemniscate would be the apex
direction. See Figure 12 (page 15). The earth is reproducing the lemniscatory motion of the sun.
- If one includes in the considerations a rhythmic pivoting motion of the Earth-Sun lemniscate, one obtains a completely different trajectory of the earth in the universe. In addition, it comes about within this frame of reference that the paths of the sun and the earth each run in their own areas of space. Both areas of space only touch at one point, which is traversed twice a year by the sun and the earth. See Figure 70 (page 49).
- Figure 70 (page 49) shows a purely planar representation. Including the advancing of the lemniscate of the sun, one obtains the three-dimensional sun movement shown in Figure 72 (page 50), a helical motion with S-curvy parts. As shown in Figure 73 (page 51), the earth performs movements that can be called artistic movements.
- If one adds to the rhythmic pivoting motion of the Earth-Sun lemniscate a continuous rotary motion of the same, as can be seen in Figures 74 to 86 (from page 52), the trajectories of the sun and earth will yield straight lines, with their inclination angles possibly corresponding to the edge lines of the Platonic bodies. Please refer Figure 87 (page 58) and Figure 88 (page 59).
- Taking into account additionally that the entire, rhythmically pivoting and rotating Earth-Sun lemniscate is taken along with Mars on its lemniscatory path, you will again obtain a changed course of movement.
- A further change of the spatial motion of the earth and the sun is obtained by granting the lemniscate of Mars an own rhythmic pivoting motion, as it must be assumed for the Earth-Sun lemniscate. To be able to depict this, further considerations are needed.
- The ultimately resulting, actual space movement of earth and sun will only be preserved, if even the lemniscates of Jupiter and Saturn with their pivoting motions are included in the considerations, with the simultaneous assumption that Saturn may be the outermost planet of the whole system of lemniscatory paths.

All in all, we can state that the question of the actual path of the earth in space cannot be answered so easily. The question as such would have to be formulated in a different way: What orbit does the earth have in space when one assumes that the sun is stationary, when one takes into account the individual movement of the sun, when one takes into account a pivoting or rotary motion of the Earth-Sun path, etc. etc.? This makes Rudolf Steiner's statement, already quoted at the end of section 1.1 of these considerations, even more understandable: "... the matter itself is so extraordinarily complicated that one can actually only reach the schematic conceptions".

Finally, one statement of Rudolf Steiner still remains to be explained: "You cannot draw into the same space the path of Venus and the path of Saturn. From this you can see ... that it is not possible to draw a solar system." (Lecture from 2 May 1920 [5]) Here Venus stands for the inner planets and Saturn for the outer planets. - What does it mean "cannot draw into the same space"? The answer follows from Steiner's remark: "Mercury, Venus, earth, follow the sun, and these three: Mars, Jupiter, Saturn go ahead." (Conference of 25 September 1919 [4]). The "going ahead" includes an in-motion setting - a drawing forward of the entire inner planetary system. It follows the outer planets as they travel through space. While the laws of the inner planetary system can be graphically represented on a stationary system, because the inner planets run on heliogeocentric path, i. e. oriented towards the Earth-Sun lemniscate as a center, in such a system one can draw the outer planets at best at their correct distance to the sun, but the planets themselves will not be at the specified place. If the real paths of the outer planets are to be drawn with their real locations, the entire inner planetary system must move with every movement of the outer planets.In contrast to the orbits of the inner planets, the orbits of the outer planets are not oriented towards the center, but towards the periphery, toward the perimeter. They travel, led by Saturn, on a large helical path, presumably caused by a pivotal motion of Saturn's lemniscate, as shown for
the sun's helical motion in an advancing lemniscate (Figure 72, page 50). The Copernican system can only provide projections of the positions of the outer planets, because these are imagined as running heliocentrically like the inner planets. Instead of allowing the sun to travel with the outer planets, they are drawn so far to the sun that the sun itself can remain unmoved. The antagonism of the situation in the inner and outer planetary system is completely disregarded in the Copernican system. But to open up this precisely is what humanity needs as the next step, in the spirit of Rudolf Steiner's words: "Now you have the external image, the purely geometric external image; the other image will be added, and only from the union of the two images will later humanity gain the idea that it must have. " (Lecture from 1 October 1916 [1])

### 1.5 Summary of the results

The above considerations lead to the following results:

1. An approximation to the real motion processes of the planets can be done using different calculation methods. While in Ptolemaic times one was still aware of the difference between calculation model and reality, in today's Copernican system we equate calculation model and reality.
2. Mercury's Copernican-Keplerian computable path is similar to a lemniscatory trajectory (Figure 2, page 6). The elongated S-curve in the computable Venus orbit of 2004 is similar to the transition from one lemniscate half to the other (Figure 51, page 34). Both trajectories consistently indicate lemniscatory path behaviour.
3. Mercury's sudden changes of direction, which modify its loops to acute angles, arise in the lemniscatory system quite naturally by the change of direction of the sun. They can be regarded as indicators of the natural motion of the sun. The Copernican system transmits the proper motion of the sun to Mercury's path and must therefore introduce three exception rules: high eccentricity, high orbit inclination and perihelion precession. Also for the path of Venus a high orbit inclination has to be introduced. In this way, the motion sequences can be reconstructed purely computationally with an elliptical orbit model. An epicyclic calculation model, however, is closer to the movement of the inner planets along the Sun's lemniscate path.
4. A lemniscatory trajectory of sun and earth can be reconciled with the apparent circular path of the sun in the sky:
a. when the common Earth-Sun lemniscate performs a rhythmic, semi-annual pivoting motion of $260^{\circ}\left(3 \times 60^{\circ}\right.$ and $\left.2 \times 40^{\circ}\right)$ in one direction and then in the opposite direction. The resulting paths of sun and earth can be described as "S-curve helices".
b. when the entire inner planetary system is one unit - i.e. all planets bound to the EarthSun lemniscate, including the sun - and performs a continuous annual $360^{\circ}$ rotation. In the Copernican System this rotary motion is transmitted to Earth in the form of an annual orbit of the Earth around the Sun. In the lemniscatory system, the annual rotation of the inner planetary system results in "straight-linear path sections" of the sun and the earth. According to Steiner, these should follow the lines of the Platonic solids. To be able to judge the inclination angles of the straight-linear path sections, it is first necessary to consider the vertical movements of the sun and earth in the advancing lemniscate more closely.
c. when the axis of the advancing common Earth-Sun lemniscate undertakes various compensatory movements to ensure that the earth's axis always points to the celestial pole and the sun's path is always at $0^{\circ}$ ecliptic latitude. According to Steiner, the required axis movement itself is lemniscatory.

An answer to the still unanswered questions concerning the vertical movements of the sun and the earth in the advancing lemniscate, the angles of inclination of their straight-linear path sections and the lemniscatory axis movement is sought in the following PART 2 of the considerations.

## PART 2

Summary: After a brief look at the evolutionary stages of the astronomical worldview, first the distance problem between the Sun and the Earth during the change of lemniscate halves will be discussed. A solution is provided by temporarily separate pivoting motions of the lemniscates of sun and earth. This results in perfectly straight-linear paths of the sun and the earth, which form a cosmic crossroads. The center of the ecliptic performs a circular path. Cross-path and circular make up a Celtic cross. Also discussed are: the need for the third Copernican law in the lemniscatory path system, the upward movements of the Sun and Earth on their advancing lemniscates, the lemniscatory movement of the lemniscates axis and the effects of the movement of the sun towards the apex on the course of lemniscatory paths.

### 2.1 Stages of development of the astronomical worldview

Modern scientific astronomy is certainly of the opinion that the Copernican-Keplerian elliptical orbit system has finally grasped the reality of planetary motion. But if you look at the history of astronomy, it turns out that every age, every epoch of culture, has developed a world view that seems to be particularly suitable for them, and has for some time considered it to be the right one. It is all too easily overlooked today that we too are far from reaching the end of human development. We must assume that future ages and cultures will also develop and appreciate their own worldview just as much as we value ours today and that future worldviews will differ equally fundamental from our own today as the world-views of former cultures.

Rudolf Steiner divides the post-glacial (post-Atlantean) cultural development of humanity into five cultural epochs. The first, the ancient Indian epoch, began towards the end of the Ice Age, around the 8th millennium B. C. At that time, people still lived so strongly in the soul, in a state of ancient dreamlike clairvoyance, that they perceived the physical outside world as unreal, mere illusion (maya). A memory of this ancient state of consciousness has been preserved among the natives of Australia. In this regard, they speak of the "dream time" in which all people once lived. Such a culture, completely devoted to the inner life of the soul, naturally had little reason to develop an outer world view.

The earliest cosmology developed in the following ancient Persian culture under the leadership of the great Zarathustra. He aroused people's interest in the perception, observation and appreciation of the outside world. Zarathustra pointed out that soul events, the actions of spiritualpsychological beings that could be encountered in the dreamlike old clairvoyance, are depicted in the celestial phenomena of the physical outside world. According to the hierarchical order of these beings, he divided the cosmos into seven planetary spheres, the Moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn, surrounded by the twelve zodiac images. As a reflection of psychological events, heaven was not subject to the physical laws of the earth. Heaven and earth were in opposition to each other. They were an expression of the dualism of light and dark forces. Until the beginning of the 3rd millennium B. C., the ancient Persian culture left its mark on the development of mankind.

In the subsequent Egyptian-Chaldean-Babylonian culture (2907-747 B. C.), the strict separation of heaven and earth was maintained. It basically remained in existence until the 16th century. Although they began to systematically observe the movements of the sun, moon and planets and to examine them for regularities, the focus was mainly on the temporal events related to the inner
experiences of the soul. For example, early scientific observations have shown that the moon performs a cycle of movement in 28 days, Mercury in 88 days, Venus in 225 days, the Sun in one year, Mars in about 2 years, Jupiter in 12, and Saturn in 30 years. This chronological order and order of the planetary world was still in harmony with the order of heaven taught by Zarathustra in the preceding cultural epoch.

In the Greco-Latin cultural period (747 B. C. to 1413 A. D.), humanity continued to live more in the physical outer world. They began to condense the spiritual-psychological spheres to transparent "crystal" spheres, all of which were thought to be in the same distance from each other. The planets were attached to these spheres. The planetary movements came about by rotation of the individual spheres, with all crystal spheres rotating at the same speed, and some planets only ran faster because their spheres were smaller and closer to Earth. The idea of a celestial mechanics emerged. However, in the attempt to calculate the planetary positions in advance for astrological purposes, the loop formation of the planets with times of forward and backward movement caused great problems. They contradicted the required steady motion and could not be grasped mathematically. Since the perfection of the cosmos in the Aristotelian sense corresponded only to the spherical form of the spheres and circular motions, Ptolemy tried to solve the problem of planetary loops through the interaction of several circular motions. He assumed that the planets were not attached directly to the spheres, but on small, rotating circles (epicycles) that run along the spheres with their center. In this way, planetary positions for the needs at that time could be calculated quite well.

With the addition of epicycles by Ptolemy a spatial process is inserted into the world view for the first time. However, a real understanding of the spatial depth was not yet available. Ptolemy also thought the spheres equidistant from each other and moving at the same speed. Likewise, he maintained the purely temporal principle of order for the order of the planets. The Ptolemaic epicyclic system is therefore a world view mixed temporally and spatially.


Temporal order of the planets

| Moon | Mercury | Venus | Sun | Mars | Jupiter | Saturn |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 28 days | 88 days | 225 days | 1 year | ca. 2 years | 12 years | 30 years |

Temporal-spatial worldview
Figure 90: Ptolemaic epicyclic system for explaining planetary loops [8]

Humanity did not enter in its consciousness entirely out into space until the beginning of the fifth cultural period in the Renaissance.* Through the discovery of the principle of the central perspective, it became possible to grasp the spatial depth, to draw and paint it. At the same time as Leonardo da Vinci, Michelangelo and Raphael introduced the perspective into their art, Copernicus was the first to attempt to apply the perspective laws of earthly space to the celestial phenomena and to understand the looping of the planets not as a real movement but as purely spatial-perspective phenomenon. For this, however, the sun had to rest and the earth had to be set in motion. Also the distances of the planets to each other and to the sun had to be assumed as different. The planets no longer ran forwards and backwards, but the earth moved forth and back on a circular orbit around the sun, with its own motion causing the only apparent forward and backward movement of the outer planets. Mercury and Venus had to swap their places in the order of the planets in this new, purely spatial order.


Figure 91: The Copernican explanation of the looping of the planets

But Copernicus needed some modifications to get correct predictions. He also had to add epicycles. However, much smaller epicycles than Ptolemy's were sufficient. Also, according to his calculations, the sun was not exactly in the middle of the planetary spheres. Since Copernicus wanted to maintain the Aristotelian demand of pure circular motion, he introduced eccentric circles.** All in all, his system thus allowed much better explanations of the planetary motions as well as calculations of the planetary positions in advance and back in time.

[^7]The new heliocentric model also allowed a more accurate determination of the length of the year, so that the tables of Copernicus became the basis for the Prutenic tables, which were included in calendar reform carried out in 1582 under Pope Gregory XIII. Above all, the church appreciated the mathematical possibilities of the new system. It was seen as a method of calculation and not as reality. In that sense, the new system initially did not pose a threat. Opposition initially came mainly from Luther and Melanchthon. They fiercely resisted the assumption of a moving earth and a resting sun because they saw in it a contradiction to the wording of the Bible. According to Joshua 10:12, God said, "Sun, stand still at Gibeon, and moon in the Valley of Aijalon. And the sun stood still, and the moon stopped." But if God commanded the sun to stand still, it must have moved beforehand, and consequently, instead of the sun, rest the earth. Only decades after the urgently needed calendar reform, a hostile attitude towards heliocentrism developed in the Catholic Church. In 1616 (73 years after Copernicus' death), the Inquisition put all heliocentric-oriented works on the Index of Forbidden Books and persecuted researchers such as Galileo Galilei and Giordano Bruno. In this way, however, the general recognition of heliocentrism as a spatial reality could only be delayed, not prevented. Kepler's elliptical orbits, Galileo's laws of gravity and the abolition of the fixed star sphere in favour of Giordano Bruno's idea of an infinite space ultimately prevailed. From now on, all physical laws observed on earth have been fully applied to the celestial phenomena also. The fundamental separation between heaven and earth has been lifted. The sky was "made terrestrial".

In the ensuing period, mankind practiced itself in making perspective considerations of planetary motion on the basis of a resting center. The next step of developmental was the realization that in the realm of the planetary everything is in motion. Neither the earth nor the sun rests. They're both moving. Since the 19th century, natural science has assumed a straight-linear movement of the sun to a target point in the constellation of Hercules (Solar apex). The orbit of the earth around the sun changes thereby to a helical movement around the sun.

According to Rudolf Steiner, however, the planets would not move on circular orbits, but on lemniscatory paths. The paths themselves carry out additional motions, like a sketch in his notebook suggests.* It is only through the interaction of all movements that the sun's apparent course of the year on a circular path through the zodiac constellations results. As possible movements of the lemniscate-paths the following come in question: e.g. a rhythmic pivoting motion, an even rotary motion, an axis movement, which in itself, according to Rudolf Steiner, is again lemniscatory. In addition, the entire lemniscatory path system of the inner planets, which seems to form a unity in itself, is to be guided by the outer planets in their run on their own moving lemniscatory paths through space. If you want to find and describe the laws that exist in such a complicated system, you have to set your thinking in motion far more than we are used to today. For each consideration, a suitable fixed point must be sought in order to obtain a valid perspective view from there. In this way different and complementary partial views of a complicated motion structure can be comprehended step by step. Perhaps there are ultimately twelve valid perspectives for describing the planetary movement, similar to the twelve worldviews described by Steiner.** At the end, the task remains to merge all sub-views into one unit in mind.

The very next step in the development of humanity's consciousness is quite obviously to evolve beyond the sensory world, to penetrate the "veil of senses." Copernicus has already taken the first step by assuming a resting of the sun and a movement of the earth against the sensory experience, which is not to be discussed, and which shows exactly the opposite picture. The next step could be to understand the rhythmic motions of the planets as a physical image of etheric formative forces, for the basic characteristic of all living things is rhythm, rhythmic effects of forces, which can be portrayed in fluent movements in a sensorial way. So far, astronomy was

[^8]based only on circular orbits or elliptical orbits very similar to a circular orbit. Rudolf Steiner adds two other, in principle different, path courses: lemniscatory and straight-linear. In this way he describes the three basic forms according to which the three-parted human body is also built: The head is formed according to the principle of the circle (or sphere). The two-part circulatory system with the heart as a crossing point is based on the principle of the lemniscate. The limbs are formed according to the principle of straight lines. In this way, the human being becomes a microcosmic image of the macrocosmic forces.

### 2.2 Continuation of the considerations to the straight-linear path sections of sun and earth

In PART 1, in Section 1.3.5, "Pivoting motion of the Earth-Sun lemniscate with simultaneous rotary motion in the context of the system rotation" (published in JUPITER, Vol. 5, No. 1, September 2010) it was described how the trajectories of sun and earth on a lemniscatory path ultimately lead to straight-linear paths when the lemniscate pivots in half-yearly rhythm, first counter-clockwise and then clockwise, and at the same time an annual continuous rotary motion of $30^{\circ}$ per month takes place in clockwise direction. Apart from the monthly positions of the sun and the earth, a number of half-month positions were also determined, the latter, however, only for the first half of the year and as samples, because the printing of the above-mentioned JUPITER edition was imminent, as well as the conference "Earth, Sun and the Lemniscate"" in Dornach (October 2010), which required time-consuming proofreading and preparation work for the conference. In the meantime, the remaining examination steps with regard to the half-month positions of sun and earth have been made up for. As a result, they have confirmed straight-linear path sections, but only for one half of the year. In the graphical determination of the half-month positions, the inclination of the earth's path against the sun's path by $23.5^{\circ}$ was also taken into account. In Figure 92, one looks from the ecliptical pole vertically down to the plane of the ecliptic, in which the sun moves along its lemniscatory path (orange) according to Rudolf Steiner. As the earth's path is inclined to the sun's path, the former appears shortened (blue). It is to be imagined in such a way that in the upper half of the picture it tilts towards the viewer out of the paper surface and tilts away from the observer in the lower picture surface, running quasi underneath the paper surface. The (slightly compressed) course of the earth's lemniscate, which has changed in perspective to the sun's lemniscate, results in only minor shifts for the already determined monthly positions.

Including the (subsequently determined) remaining half-month positions of a year results in bow-curved path sections in addition to the straight-linear path sections for a part of the year. Figure 93 shows, in addition to the already determined positions at monthly intervals (sun yellow, earth blue*), the positions at half-monthly intervals (white: sun big, earth small). For the period from 22 December until 20 March this confirms the strictly straight-linear course. After that, the sun deviates slightly from the strict straight-linear movement ( 20 March, in the middle of the picture, until 21 May). From 21 May until 23 August the path of the sun runs almost semicircular and then turns into a large loop, which finally ends at an acute angle at the starting point of 22 December. The earth carries out this movement in reverse order. After a strictly straight-linear path section from 22 December until 20 March it runs a loop until 22 July and then goes into a nearly semicircular path until 23 October. This is followed by a path section deviating from the straight line with only slight momentum from 23 October to 22 December.

[^9]

Figure 92: View from the ecliptic pole pointing vertically down to the Sun's lemniscate with Earth's lemniscate tilted by $23.5^{\circ}$


Figure 93: Straight-linear and bow-curved path sections of the sun and earth

### 2.2.1 The Problem of the lemniscate half change

At this point it is necessary to point out a still unsolved problem, which has remained disregarded in all previous considerations and could also be temporarily disregarded, because it was first and foremost about to understand the course of a lemniscatory motion of the sun and the earth in their fundamentals and to bring it into harmony with the circular path of the sun through the constellations of the zodiac. If you look at those figures in PART 1 which show a lemniscate half change by sun and earth*, you can clearly see that the distance between sun and earth has to increase, when the sun changes into the other lemniscate half and the earth still wants to keep the given monthly positions on its lemniscate. Figure 94 shows the situation immediately before changing the half. Here the distance Earth - Sun corresponds to the diameter of a lemniscate half.


Figure 94: The positions of the sun and the earth immediately before the sun enters the other lemniscate half

If the sun advances to its next month's position, followed by the earth, the latter can only occupy its given monthly position on the lemniscate if the distance to the sun increases significantly (see Figures 95 and 96).

Only when the earth on 21 June is arriving at the lemniscate's midpoint, its distance from the sun is again equal to the diameter of the lemniscate half (see Figure 97).

In the Copernican system, too, there is a rhythmically changing distance between the earth and the sun (perihelion and aphelion). However, this is not nearly as big as it must be assumed for the depicted lemniscatory motion.

Figures 60 to 62 (from page 44)


Figure 95: The positions of the sun and the earth in the course of the sun's lemniscate half change on 20 April


Figure 96: The positions of the sun and the earth in the course of the sun's lemniscate half change on 21 May


Figure 97: The positions of the sun and the earth on 21 June
Consequently, the question arises as to whether the path positions in the course of a lemniscate half change are quite different than previously assumed. Rudolf Steiner expressly emphasizes that we are "dealing with a succession of the earth towards the sun, in a manner of a hurrying ahead of the sun and a following of the earth." (Lecture from 12 January 1921) This can only be understood in this way that the sun pulls the earth behind it at an almost constant distance. The consequence of this would be that the earth would have to run much faster on its lemniscatory path for some time and to cover a much greater distance on its path than previously assumed in order to be able to maintain the distance to the sun.


Figure 98: The positions the sun and possible positions of the earth on 20 April and 21 May

As Figure 98 shows, the earth would have to cope from 20 March to 20 April with even double the monthly distance, i.e. it would have to take up the position, which is actually intended for 21 May. In addition, the pivoting motion of the lemniscate would have to be different, so that the sun can maintain its positions in the zodiac, because the connecting line between sun and earth rotates clockwise by about $18^{\circ}$. The position of the earth on 21 May, however, would have to move closer to the position of 21 June (lemniscate midpoint), so that the earth would have to run much slower in May and June, in order not to change too early into the other half of the lemniscate. Such enormous variations in speed cannot be assumed to be real, even if they are imperatively demanded by a lemniscatory motion for geometric reasons. But how can the problem be solved without giving up a lemniscatory movement?

### 2.2.2 Separated pivoting motions of the lemniscates of the sun and the earth

The pivoting motions of the lemniscate are a compelling consequence to allow the annual circular path of the sun through the zodiac constellations. One of Rudolf Steiner's surviving sketch can be regarded as confirmation of such a pivoting motion of the common Earth-Sun lemniscate. In addition to a self-contained lemniscate, the sketch shows half a lemniscate, which apparently makes a counter-clockwise pivoting motion in relation to the self-contained lemniscate. See Figure 99. But now the sketch can be interpreted in a different way. Possibly the self-contained lemniscate does not represent the common Earth-Sun lemniscate, but only the Earth lemniscate. The half lemniscate, which dissolves and pivots away from it, would then be a section of the Sun lemniscate. That would explain why the small lemniscate of an inner planet is only tied to half the lemniscate and participates in the pivoting motion, for the movements of the inner planets are bound to the movement of the sun. The section of the Sun lemniscate that pivots to the left remains in its lemniscate midpoint firmly connected to the self-contained Earth lemniscate, so that Rudolf Steiner could nevertheless simply speak of a common "Earth-Sun path".


Figure 99: Rudolf Steiner's sketch from notebook 121

In the following it shall be shown, how the problem of the distance Earth - Sun during the lemniscate half change can be solved with the help of separate pivoting motions of both lemniscates. Figure 100 corresponds to Figure 74 (page 51) in PART I. There, the lemniscates of the earth and the sun are still combined to a common lemniscate. In the now following figures,
however, they are shown separately just in order to be able to take into account the inclination of the earth's path in relation to the sun's path. The view goes from the ecliptical pole vertically down to the plane of the ecliptic in which the solar lemniscate lies. The blue Earth-lemniscate appears a bit shorter due to its inclination. The upper part (thick blue circle) is to be imagined in such a way that it protrudes above the paper surface and inclines towards the viewer. The lower part (thin blue circle), on the other hand, is to be thought of as running underneath the paper surface. It leans away from the viewer. The straight-linear sections of the first three month's movements, which have already been determined and described above, are marked as yellow and light blue stripes in the background. The sun and the earth move along these stripes in the three winter months, each on its own straight-linear path. During this time, both lemniscates perform a common pivoting motion of $-60^{\circ}$ per month (counterclockwise). In order for the starry sky to be able to rest, the double system of the Earth-Sun lemniscate additionally has to perform a continuous rotary motion of $+30^{\circ}$ per month. For the figures 100 to 103 , this results in a pivoting motion of apparently only $-30^{\circ}$ per month.


Figure 100: Position of the Earth-Sun lemniscate with resting starry sky on 22 December
From winter solstice to vernal equinox, the sun and the earth move in the same lemniscate half and thus can easily maintain their mutual distance (= approx. the diameter of one lemniscate half). From 20 March, however, the sun changes to its summer half lemniscate (Figure 103, orange, thin circle) and pulls the earth behind it. Its distance to the earth can be maintained only if both lemniscates go "separate ways" for the duration of the change, as indicated in Rudolf Steiner's sketch. In the figure, it is noted that in the three spring months, only the earth's lemniscate will continue the previous - $60^{\circ}$ (counterclockwise) pivoting motion by - $60^{\circ}$ (counter-clockwise) per month. The sun's lemniscate is resting instead. Due to the simultaneously happening continuous rotary motion of the double system, in Figure 104 the sun's lemniscate nonetheless appears to be pivoted by $+30^{\circ}$ and the earth's lemniscate only to be pivoted by $-30^{\circ}$. This results in a surprising result. Although the sun and the earth consequently continue their way on their own lemniscatory paths, both of them do not only keep the previous distance from each other but also their straight-
linear motion. The earth begins to run back the same way it did in the winter months (light blue stripe). On 20 April it reaches the position it already held on 19 February. The sun, on the other hand, continues its straight-linear path course downwards (extension of the yellow stripe).


Figure 101: Position of the Earth-Sun lemniscate with resting starry sky on 21 January


Figure 102: Position of the Earth-Sun lemniscate with resting starry sky on 19 February


Figure 103: Position of the Earth-Sun lemniscate with resting starry sky on 20 March


Figure 104: Position of the Earth-Sun lemniscate in relation to each other with resting starry sky on 20 April


Figure 105: Position of the Earth-Sun lemniscate in relation to each other with resting starry sky on 21 May


Figure 106: Position of the Earth-Sun lemniscate in relation to each other with resting starry sky on 21 June

Figure 106 shows that the Earth lemniscate, after three pivoting motions of each $-60^{\circ}$ has turned by a full $180^{\circ}$ at the summer solstice on 21 June. The winter-lemniscate-half of the earth (blue, thick circle) is now in the lower half of the picture together with the summer-lemniscate-half of the sun's lemniscates (orange, thin circle). The summer-lemniscate-half of the earth (blue, thin circle) is in the upper half of the picture together with the winter-lemniscate-half of the sun (orange, thick circle). The earth is now changing to its summer-lemniscate-half. However, this changing the halves has no effect on the distance to the sun, because due to the $180^{\circ}$ pivoting performed by the earth's lemniscate now sun and earth are running in the same direction (to the top right). A comparison of Figures 104 and 105 clarifies how this situation comes about.

From 21 June until 23 September neither the earth's lemniscate nor the sun's lemniscate perform any pivoting motion. Both, after they have reunited, are further rotated together by the continuous rotary motion of the double system of $+30^{\circ}$ per month (Figures 107 to 109). From 22 July it becomes apparent that the earth's horizontal straight-linear path (light blue stripe) will form a large cosmic cross with the vertical straight-linear path of the sun (yellow stripe). As the sun begins to move back on its previously completed path, the earth continues its course to the right.


Figure 107: Position of the Earth-Sun lemniscate with resting starry sky on 22 July
On 23 September (Figure 109) the earth reaches the outermost point of its horizontal path (light blue stripe). Now it returns to the lemniscate midpoint. The sun changes over to its winter-lemniscate-half (orange, thick circle). It seems that the sun and the earth are now paradoxically running towards each other (small arrows, orange and blue). The distance between the sun and the earth, however, will be maintained because both lemniscates now again go "separate ways" for the duration of the change (Figure 110). In the autumn months, only the sun's lemniscate performs the well-known pivoting motion of - $60^{\circ}$ per month. The earth's lemniscate is resting instead. Nevertheless, it appears to be pivoted by $+30^{\circ}$, because the continuous rotary motion of the double system takes place at the same time. However, the angular distance between the longitudinal axis of the sun's lemniscate and the longitudinal axis of the earth's lemniscate is the aforementioned $60^{\circ}$. This situation corresponds exactly to that sketched by Rudolf Steiner on notice sheet No. 121
(Figure 99, page 74). The sun's lemniscate detaches from the earth's lemniscate and pivots away from it counterclockwise. The paths of Venus and Mercury (not shown in Figure 110) are bound to the sun's path and therefore must necessarily complete the pivoting motion of the sun's lemniscate.


Figure 108: Position of the Earth-Sun lemniscate with resting starry sky on 23 August


Figure 109: Position of the Earth-Sun lemniscate with resting starry sky on 23 September


Figure 110: Position of the Earth-Sun lemniscate in relation to each other with resting starry sky on 23 October


Figure 111: Position of the Earth-Sun lemniscate in relation to each other with resting starry sky on 22 November


Figure 112: Position of the Earth-Sun lemniscate with resting starry sky on 22 December


Figure 113: The cosmic cross path of Sun and Earth

The result of the above considerations, the straight-linear trajectories of Sun and Earth over the course of a year, is summarized in Figure 113. The equinox positions of the sun and the solstice positions of the earth together form the midpoint of a large cosmic cross path. The solstice positions of the sun form the endpoints of the vertical arms of the cross. The equinox positions of the earth form the endpoints of the horizontal arms of the cross. This creates a large, almost isosceles cross. The view goes from the ecliptic pole vertically downwards. Due to the inclination of the earth's path in relation to the sun's path, the horizontal cross arms appear a little shorter than the vertical ones. Figure 113 also shows the Copernican conception of the earth's path (dashed in light blue). Ultimately, a structure emerges that is startlingly reminiscent of a Celtic cross and
raises the question: did the ancient Druids know about these relationships? Surely they could not intellectually know about it. But it is quite possible that they "knew" it imaginativelyinspirationally and immortalized this insight in their stone crosses, where both side arms and the headboard are of the same length.

Rudolf Steiner's seemingly contradictory statements about the three fundamental forms of planetary movements, the circular, lemniscatory and straight-line movements, are ultimately confirmed much better than would have been expected from the initial considerations. Finally, the laws underlying the pivoting motions of the lemniscates of the Sun and Earth can be described as follows: Each season has its own variant of motion, which is consistently maintained for three months. If a pivoting motion takes place, it is always - $60^{\circ}$ (counterclockwise). In winter it is carried out by both lemniscates together, in spring only by the earth's lemniscate and in autumn only by the sun's lemniscate. Both lemniscates rest in summer and there is only the year-round continuous rotary motion of the entire inner planetary system of $+30^{\circ}$ per month.

| Winter | Spring | Summer | Autumn |
| :---: | :---: | :---: | :---: |
| Common <br> pivoting motion <br> of both lemniscates <br> by $-60^{\circ}$ | Sole <br> pivoting motion of <br> the earth's lemniscate <br> by $-60^{\circ}$ | Common resting <br> of both lemniscates | Sole <br> pivoting motion of <br> the sun's lemniscate <br> by $-60^{\circ}$ |
| Year-round continuous rotary motion of $+30^{\circ}$ per month (zodiacal month) |  |  |  |

### 2.3 The circular path of the midpoint of the ecliptic

Rudolf Steiner said in the lecture of 29 April 1908 [10] about the ecliptic: "What is called the obliquity of the ecliptic is the line of gravity between the sun and the earth. One has forgotten that the earth revolves once around the axis of the ecliptic over the course of the year." The first sentence wavers our present conceptions of the oblique plane of the ecliptic and the second sentence wavers our present conceptions of the movement of the Earth.

The ecliptic is a phenomenon based on geocentric observation. Viewed from the earth, it looks as if the sun would perform a circular path through the twelve zodiac constellations over the course of a year. This circular path shows a sloping due to the inclination of the Earth's path in relation to the sun's path: the "obliquitiy of the ecliptic". Rudolf Steiner's statement "What is called the obliquity of the ecliptic is the line of gravity between the sun and the earth" means that there is no sloping circular path, i.e. no circular ecliptic plane. He claims that there is only an ecliptic line, the connecting line or "gravity line" between the sun and the earth.

The second sentence about the axis of the ecliptic is equally puzzling. Where is this axis of the ecliptic around which the earth is supposed to revolve in the course of a year? Geocentrically, the axis of the ecliptic runs through the center of the earth, but is tilted by $23.5^{\circ}$ relative to the axis of the earth. Should the earth revolve around both axes at the same time? How should that be possible? The lemniscatory path system offers a solution here.

Due to the pivoting motions of the two lemniscates, the reversal of the sun's direction in the lemniscate is cancelled. But if only these swinging motions were present, the sun, viewed from the earth, would always remain at the same position in the zodiac, because a kind of equilibrium state, a status quo, arises. Only through the continuous rotary motion of the complete inner planetary system is the connecting line between the sun and the earth rotated by $+30^{\circ}$ per month (see figures 74 to 86 in PART 1, starting on page 51). Seen from the earth, it looks as if the sun is moving in a circular path in the sky in the oblique ecliptic plane. In fact, there is only this connection line between the sun and the earth. This is apparently what Rudolf Steiner means when he says: "What is called the obliquity of the ecliptic is the line of gravity between the sun and the earth."

The axis of the ecliptic in the lemniscatory path system runs vertically from top to bottom (from the ecliptic pole) through the midpoint of the connecting line or "gravity line" of the sun and the earth, i.e. in about 0.5 AU distance from both. In the course of a year both (!) turn around this center as if they were running on a common circular path one behind the other. Their connecting line corresponds to the diameter of the common circular path. Figure 114 shows how the connecting line between the sun and the earth rotates $+30^{\circ}$ per month, from $0^{\circ}$ Capricorn (green line [with a green earth and green sun]) to $0^{\circ}$ Aquarius (blue line [with a blue earth and blue sun]) to $0^{\circ}$ Pisces (violet line [with a violet earth and violet sun]), etc. But beyond that, it moves as well, whereby the midpoint of the ecliptic or the axis of the ecliptic migrates from the center to the lower left (red dots on the green, blue and purple connecting lines between sun and earth). For this reason, at first glance no common circle path of sun and earth around the midpoint of the ecliptic becomes visible.


Figure 114: Annual rotation of the connecting line Earth- Sun by $+30^{\circ}$ per month and migration of the midpoint or the axis of the ecliptic

In fact, however, the common circular path of the earth and the sun runs like an epicycle along an equally large central circular path (red circle in Figure 115). The connecting lines between the sun and the earth intersect halfway on the red circular path. The epicyclic movement is depicted for the first three months (green, blue and violet). You can see how the epicycle rotates by $+30^{\circ}$ per month (clockwise), together with the sun and the earth, and that the midpoint of the epicyle (= midpoint of the dotted lines) always lies on the red circular path. Because of the inclination of the earth's path towards to the sun's path, the epicycle is subject to inclination. It must therefore be minimally elliptically deformed when projected onto the ecliptic plane. The central (red) circular
path also becomes an ellipse. It is a little less wide than high. In the figure, however, this is hardly noticeable visually.


Figure 115: The epicyclic movement of the common circular path of Sun and Earth along the circular path of the midpoint of the ecliptic

If one wishes to summarize these two movements in two sentences, the following words of Rudolf Steiner result: "What is called the obliquity of the ecliptic is the line of gravity between the sun and the earth. It has been forgotten that the Earth revolves once around the axis of the ecliptic over the course of a year." The second sentence could be extended: "... the earth and the sun revolve once around the axis of the ecliptic over the course of the year."

Adding the circular path of the midpoint of the ecliptic to the cosmic cross path of the sun and the earth, the result is even more like a Celtic cross with equally long cross arms extending beyond an inner circle (Figure 116).


Figure 116: The cosmic cross path of Sun and Earth with the circular path of the midpoint of the ecliptic

### 2.4 The third Copernican law as a necessary component of the Lemniscatory Path System

An annual continuous rotation of the Earth-Sun lemniscate or the entire inner planetary system by $+30^{\circ}$ per month (zodiacal month) also causes a rotation of the earth's axis. When everything that is in the inner planetary system is rotated, the axes of the world bodies in it are also rotated. In order to align the earth's axis firmly with the celestial north pole and thereby to enable the specific seasonal positions of the earth's axis towards the sun, an additional rotation is required that cancels out the annual rotation of the earth's axis. Rudolf Steiner also says, "that the earth revolves once around the axis of the ecliptic over the course of the year." (Lecture of 29 April 1908 [10]) This can basically only be understood as a rotation of the Earth's axis, because the otherwise known rotation of the earth is not annual but daily and not around the axis of the ecliptic but around the Earth's axis. Apparently there is an additional annual rotation of the Earth, where the Earth's axis describes a circle or a cone in the sky. Such a movement was already described almost five centuries ago by Copernicus as the "third movement of the Earth". Rudolf Steiner usually called it "the third Copernican law".

Today, we find it difficult to comprehend the third Copernican law. We assume that the planets would follow an original impulse of movement and would hurry away from the sun (centrifugal force) if the gravitational force of the sun (centripetal force) did not stop them and force them onto their orbits. At the time of Copernicus the world of ideas was a very different one. People still thought more in the Pythagorean sense. The sun was regarded as the lord and ruler of the planets, which does not simply stop them from hurrying away, but actively "leads" them on their orbits. In the 10th chapter of the first book of "De revolutionibus orbium coelestium" [17] Copernicus writes*:
"In the middle of them all, however, resides the sun. For who wants, in this beautiful temple, to place this lamp in a different or better place from where it can illuminate the whole at the same time? Some do not inappropriately call it the lamp of the world, others the spirit, yet others call it the leader. Trismegistus calls it the visible God, the Electra of Sophocles the all-seeing. The sun, just as if sitting on a royal throne, directs the family of stars, leading them in a circle. Also, the earth is not robbed of the service of the moon, but, as Aristotle said in "de animalibus", the moon has the greatest kinship to the earth. However, the earth receives from the sun and is fertilised by annual procreation."

This worldview of Copernicus, which is Christian, Gnostic and Pythagorean, finds its expression also in the fact that he used a wax seal, showing Apollo with the lyre. Apollo, the Greek god of the sun who directs and governs the planetary world, was regarded by early Christians as an image of Christ. The music of the spheres sounding from the strings of the lyre was to them a symbol of the ordering and directing power of the cosmic Word, the Logos. This shows what a deeply religious man Copernicus was and how he regarded his new astronomical findings as entirely in harmony with his own religious convictions.

[^10]According to Rudolf Steiner, Copernicus had a mission to fulfill: "Copernicus, for example, has diverted humanity from the old error that the earth was standing still. He taught that it was a mistake to assume that the earth was standing still. Kepler and Galileo elaborated on this doctrine. ... And yet both of them, Copernicus and Ptolemy, are right; it only depends on the point of view from which one looks at the sun and the earth. ... The Ptolemaic system is thus valid for the astral, the Copernican for the physical plane." (Lecture of 1 September 1906 [11]). To arrive so completely on the physical plane was precisely the task of humanity in the beginning age of the consciousness soul. But, on his own Copernicus would not have been able to accomplish this task. He was endowed for this purpose with the astral body of Cardinal Nicolaus of Cusa, a mystic and scientist, whom Rudolf Steiner described with the following words: "A glorious shining star in the sky of medieval intellectual life is Nicolaus Chrypffs from Kues (near Trier 1401-1464). He stands at the height of the knowledge of his time. He has done an excellent job in mathematics. In natural science, he may be called the predecessor of Copernicus, because he took the position that the earth is a moving celestial body like the others. He has already broken with a conception upon which, a hundred years later, the great astronomer Tycho de Brahe relied on when he hurled towards Copernicus' teaching: 'The earth is a coarse mass, heavy and unwieldy for motion; how can Copernicus turn it to a star and lead it around in the air?'"[12] Nicholas of Cusa died nine years before Copernicus was born. In another lecture, Rudolf Steiner reveals us the secret connection between the two by saying about the predecessor: "He anticipated the Copernican planetary system conception. He brought this about in a veiled way. The astral body of Nicholas of Cusa is transferred to Nicolaus Copernicus, and he describes and explains what the other had been previously given veiled. A piece of the Egyptian Hermes was contained therein [in the astral body], an important part." (Excerpt from the lecture of 25 February 1909 in Kassel in the appendix of Part I [13]) This "important piece" from the astral body of the Egyptian Hermes was of particular importance for the astronomical gift of Copernicus, for Zarathustra, the primeval teacher of astronomy, had transferred all his astronomical wisdom to Hermes, one of his most important disciples: "One [Hermes] he trained excellent in everything concerning the faculty of judgments, in the sciences, astronomy and astrology, agriculture and others. All this he transferred to this one disciple, and this was made possible by a procedure or process between them, which is a mystery. Thereby the disciple was prepared in such a way that he could wear the astral body of his teacher in the following incarnation. This reborn disciple with the astral body of his teacher is Hermes. Hermes was the great teacher and sage of the Egyptian mysteries." (Lecture of 21 January 1909 in Heidelberg [13]) However, Copernicus did not only receive a part of the astral body of Hermes and thus also of the astral body of Zarathustra, but additionally an imprint of the astral body of Christ: "In him [Nicholas of Kues] lived the astral body of Christ and this later passed into Nicholas Copernicus. Another example: the etheric body of Christ shines forth in Galileo. " (Lecture of 19 February 1909 in Leipzig in the appendix to Part I [13]) Rudolf Steiner makes clear with these statements, how those people, who are to advance the development of humanity, receive a very special equipment of components of their being from the spiritual leadership of humanity for their respective tasks. Interestingly, Copernicus did not only bear the same first name as the one whose astral body had been transferred to him, but he also had the same initials: N. K. (Nikolaus of Kues or Niklas Koppernigk*) and N. C. (Nicolaus Cusanus or Nicolaus Copernicus).

Main occupation of Copernicus was as a canon in Frauenburg** on the Baltic Sea and decreed at least on the lower ecclesiastical ordinations. He was foster son and nephew of the state bishop of

[^11]Warmia (a part of the later East Prussia), which was directly subordinate to the papal chair in Rome. The uncle would have loved to see his nephew as his successor. Copernicus, however felt an inner distance to the church. Certainly the experiences during his stay in Rome in 1500 played a role, where he could observe at close range the Vatican power and pageantry of that time under the notorious Pope Alexander VI. During his several years of study in northern Italy, he obtained the title of Doctor of the Canon Law, carried out his first astronomical studies together with his astronomy professor and completed an additional medical degree in the last three years. He then worked for seven years as secretary and personal physician of his uncle at his bishop's residence in Heilsberg*. The Pope watched his research from Rome attentively, because he urgently needed a calendar correction and was dependent on the help of all the well-known astronomers of the time. Therefore, Copernicus dedicated his main work in the preface to the then-reigning Pope Paul III (1534-1549). The insights gained from the new heliocentric view for a more accurate calculation of the year's length finally flowed into the Prussian tables, probably the most important basis for the calendar reform carried out in 1582, almost forty years after Copernicus' death (1543). The persecution of heliocentrics by the Inquisition began only in the 17th century, after the switch to the Gregorian calendar had long been completed. The false opinion, preserved until today, that Copernicus had postponed the publication of his major work out of fear of the Church's inquisition stands in complete contradiction to the historical events.

The Copernican world system emerged at the beginning of the age of intellectualism. Nevertheless, it was originally strongly influenced by a spiritual conception of the world. As already mentioned, Copernicus believed that the sun, as a guiding force, was leading the planets around. Simplified, it can be thought of as if the sun were reaching out to the earth and leading it on a circular path. By that at least the arm of the earth inevitably always remains directed towards the sun. Now, of course, Copernicus knew that the Earth does not always turn the same side of its surface towards the sun as it is the case, for example, with the moon with respect to the earth, but that the earth, in addition to its annual run around the sun, performs a daily rotation around its own axis. Independently of this, however, he imagined that the earth would tilt its axis towards the sun, and that the earth would have to maintain that attachment to the sun during the annual run, unless another, a third movement were added, which would counteract the annual rotation of the earth's axis exactly and would thus enable the continuous alignment of the earth's axis to the celestial north pole. Only this third movement made the tilting of the earth's axis towards the sun changeable, which in turn allows its fixed orientation to the celestial north pole and the phenomenon of the seasons. In his main work, in the 11th chapter of the first book under the title "Proof of the threefold movement of the earth" [17], Copernicus describes first the daily rotation of the earth, then the annual course of the earth around the sun and finally the solution to the problem of the axial position of the earth found by him:


#### Abstract

"Thus, the third movement of declination follows, also in annual rotation, but in regression, i.e. in contrast to the movement of the center [of the earth], turning backwards. And so it comes to pass through both almost identical and yet opposite movements that the axis of the earth, and hence the largest parallel circle to the equinoxes [the equator], inclines [constantly] toward nearly the same celestial region, as though they were immobile, while the sun seems to move through the obliquity of the zodiac, just as much as the movement of the center of the earth, and not otherwise, as if it were the center of the world itself, bearing in mind that the distance between the sun and the earth looked on from the fixed star sphere would escape our gaze."


Rudolf Steiner has pointed out this important insight of Copernicus on several occasions. In the lecture of 2 January 1921 he said about the problem of the alignment of the Earth's axis:
"The third principle that Copernicus asserts is that now not only does such a rotation comes about of the Earth around the North-South axis and a second around the Ecliptic axis

[^12][Earth's annual course], but that a third rotation occurs, which shows itself as a retrograde movement of the North-South axis [earth's axis] around the ecliptic axis itself. As a result, in a certain sense, the rotation around the ecliptic axis is cancelled. Thereby the Earth's axis always points to the North Pole (the polar star). While it would otherwise have to describe a circle or an ellipse around the ecliptic pole actually, as it revolves around the sun, it points through its own rotation, which is in the opposite sense - every time, when the earth moves a little further, the Earth's axis turns backward - always towards the North Pole. Copernicus has assumed this third principle that the pointing to the North Pole is made only by the fact that the earth's axis itself by rotation in itself, a kind of inclination, is continuously cancelling the other rotation. So that this does not really mean anything in the course of the year, as it is constantly being abolished. In recent astronomy, which has built on Copernicus, the peculiar thing has come to pass that one accepts the two first laws and ignores the third and disregards this ignoring of the third sentence in a manner, I would say, with a light hand, by saying: The stars are so far away that simply the axis of the earth, even if it remains always parallel, points to the same point. - So that one says: The North-South axis of the earth remains always parallel to itself during this rotation around the sun. - Copernicus did not assume that, but he assumed a continual rotation of the Earth's axis. So one does not stand on the point of view of the Copernican system, but has accepted, because it was comfortable, the first two theorems of Copernicus, omitted the third and gotten lost in the fibbing, that one does not need to assume that the Earth's axis would have to move to point at the same point, but the point is so far away that even when the axis shifts forward, it nonetheless shows on the same point. Everyone will realize that this is just a fibbing. Thus, today we have a Copernican system, which actually leaves out a very important element."

According to Steiner, especially this very important element of the annual retrograde circular motion of the Earth's axis will in the future become significant again, and it is a necessity for explaining the resting north celestial pole in the sense of the above-mentioned considerations on the straight-linear trajectories of the earth and the sun with the circular motion of the midpoint of the ecliptic around the midpoint of the lemniscate.

With the help of the retrograde circular motion of the Earth's axis, Copernicus also explained the phenomenon of precession. He assumed that the "third" movement does not exactly correspond to the "second", the annual course of the earth around the sun with a corresponding annual rotation of the Earth's axis, but that it ultimately turns out to be slightly smaller. The earth's equator is at right angles to the North-South axis of the earth. Expending it out into the universe, you get the celestial equator. This cuts the ecliptic, the apparent orbit of the sun, in the vernal point. If the annual rotation of the Earth's axis were completely cancelled out by the "third" movement, the vernal point would remain fixed at a certain zodiacal degree. If, however, the third movement is a little less than the annual axis rotation of the earth, the result is that the vernal point slowly travels backwards in the zodiac and thus explains the phenomenon of precession. Therefore, Copernicus formulated in the above already quoted section: And so it comes to pass through both almost identical and yet opposite movements that the axis of the earth, and hence the largest parallel circle to the equinoxes [the equator], inclines [constantly] toward nearly the same celestial region, as though they were immobile, while the sun seems to move through the obliquity of the zodiac." [17]

At the end of the same chapter of his major work, Copernicus points even more clearly to this connection: "So it is clear how the two opposite movements, that of the midpoint [orbit of the earth around the sun] and the inclination [backward rotation of the axis], force the axis of the earth to remain in the same inclination and in a very similar position, and that all this appears as if it were movements of the sun. We said, however, that the annual orbits of the midpoint and of the declination are nearly the same, because, if this were exactly the case, the equinoxial and solstitial points and the entire obliquity of the zodiac under the fixed star sphere could not change at all. But, since this difference is small, it does not become noticeable or only with increasing time: from Ptolemy to us, those [equinoctial and solstitial points] have receded by about 21 degrees. For this
reason some have believed that the fixed star sphere was also moving so that they therefore assumed a ninth higher sphere, and since this is not enough, now the newer ones add a tenth one, whereby they still have not reached the goal, that we aim at by the movement of the earth, which we use in the following proofs as principle and hypothesis."

Not only in the original Copernican system, but also in the lemniscatory path system the Copernican third law explains the permanent alignment of the Earth's axis on the north celestial pole and the phenomenon of precession in that the retrograde annual movement the axis of the earth is a little less than the annual continuous rotary motion of the Earth-Sun lemniscate or the inner planetary system.

### 2.5 The advancing lemniscatory paths of the Sun and the earth and the motion of the axis of the lemniscate

### 2.5.1 The upward movements of the sun and the earth

If one wants to visualize the movements of the sun and the earth in upwardly advancing lemniscatory paths, the exact course of their upward movements must first be determined. A basis for this can be the sketch of an advancing lemniscate (Figure 117) given by Rudolf Steiner (lecture of 12 January 1921, Fig. 6 [3]). Presumably, this is the advancing lemniscatory path of the sun, because in view of the gigantic size of the sun and because its path also forms the "backbone" of the inner planetary system, it is possible to assume a very even trajectory for it. Around this path Earth, Venus and Mercury would move on their own, separate lemniscatory paths.


Figure 117 - Rudolf Steiner's sketch of an advancing lemniscate [3]

Apparently, Rudolf Steiner has sketched very precisely, because a re-measurement of the angle of inclination between the end positions of the lemniscate's loops shows how close this comes to the astronomically significant $23.5^{\circ}$ (see Figure 118). This sketch thus allows for the assumption of an ascending movement of the sun on its advancing lemniscatory path at $23.5^{\circ}$ per year. The slope angle of the sun's path would then be equal to the angle of inclination of the sun's path in relation to the earth's path. The latter has also been sketched by Rudolf Steiner, but in a nonadvancing double-lemniscate, see Figure 119 (lecture of 1 October 1916 [1]). The longitudinal axis of the sun's path (brightly hatched) lies horizontally. The earth's path (hatched darker) is oblique to the sun's path. The earth climbs over and under the sun's path.


Figure 118: Inclination angle in the sketch of an advancing lemniscate by Rudolf Steiner


Figure 119: Rudolf Steiner's sketch of the double-lemniscate path of sun and earth [1]


Figure 120: Rudolf Steiner's sketch of the inclination of the earth's path to the sun's [1]

Explaining, Rudolf Steiner added: "Perspectively, you have to imagine that. If you imagine the sun's path lying in a plane, then the earth's path is lying in this plane, seen from the side. If that were the sun's path, considered as a line, the earth's path is like that." (Figure 120).

The illustrated inclination angle between the two paths is approximately $30^{\circ}$ in Figure 119 and approximately $40^{\circ}$ in Figure 120. That allows for different interpretations. Possibly Rudolf Steiner wanted to illustrate only the principle of the path inclination and not the exact angle of inclination. But perhaps he also wanted to suggest that concerning the path inclinations in a double-lemniscate, especially if this is further developed to an advancing double-lemniscate, other angles of inclination than the known $23.5^{\circ}$ come about, and possibly he was thinking of an angle of inclination between $30^{\circ}$ and $40^{\circ}$, i.e. at about $35^{\circ}$. The further considerations will show that such a path inclination comes about in the advancing double-lemniscate of sun and earth.

Using these guidelines, the positions of the sun and the earth can be determined in a nonadvancing double-lemniscate path on the solstices and equinoxes (Figure 121). The earth's axis is at right angles to the earth's path. It will not be able to maintain this position in a progressive lemniscate at first, as the Sun's path carries out a half-yearly elevation of $11.75^{\circ}$ to the right and then a half-yearly elevation of $11.75^{\circ}$ to the left, as shown in Figure 118. As a result of these elevations, both the Earth's axis and the Earth's path, as well as the ecliptic, change their inclinations in the universe. There must therefore be an additional motion of the lemniscate axis, which counteracts the uplifts exactly and thus seems to cancel them out.


Figure 121:
Positions of the sun and the earth in the non-advancing double-lemniscate on the solstices and equinoxes

The positions of the solstices can be quite easily transferred to an advancing Sun-lemniscate, because the earth crosses the sun's path at these times, so that the sun and the earth are on the path of the Sun-lemniscate that has been sketched by Rudolf Steiner. Figure 122 shows as a zig-zag line the inclination angles of the sun path taken from the sketch (Figure 118). At the winter solstice, the sun is at the left outer end of its path and at the lowest point below the earth's path (dashed in blue). The earth passes through the midpoint of the lemniscate. As the sun's path on the left side rises by $11.75^{\circ}$, the earth's path is tilted to the right by the same angle. Its angle to the vertical, which was $23.5^{\circ}$ prior to the elevation, thereby increases to $35.25^{\circ}$. Interestingly, this angle of inclination results especially at the winter solstice, just as Rudolf Steiner himself sketched with the path inclinations between $30^{\circ}$ and $40^{\circ}$ shown in Figures 119 and 120.

At the time of the summer solstice, the sun reaches the right outer end of its path and the highest point above the earth's path (Figure 123). The earth again passes through the midpoint of the lemniscate. The earth's path, tilted by $23.5^{\circ}$, is elevated together with the sun's path by $11.75^{\circ}$. This reduces the inclination of the earth's axis by just this amount to half. It is now only $11.75^{\circ}$ in relation to the vertical.


Figure 122: The positions of the sun and the earth at the winter solstice in the advancing lemniscate


Figure 123: The positions of the sun and the earth at the summer solstice in the advancing lemniscate

The positions of the sun at the equinoxes can also be easily transferred to the advancing Sunlemniscate. As Figure 121 has already shown, the Sun is then passing the midpoint of the lemniscate. On the other hand, it is more difficult to mark the positions of the earth at the times of the equinoxes next to the advancing Sun-lemniscate, because the earth is then not on the sun's path, but somewhere between the open path angles of the same. The exact position cannot be determined so easily, because the inclination of the earth's axis is variable in the advancing lemniscatory path, as it turned out. Fortunately, Rudolf Steiner has given another hint. He not only said that "the lemniscate advances", but that "its axis itself becomes a lemniscate again" (Lecture of 2 May 1920 [5]).

However, a lemniscatory motion of the axis definitely involves two opposite lateral inclinations. Figure 124 illustrates these relationships. We may assume that the maximum lateral inclinations always occur at the times of the solstices, by $11.75^{\circ}$ to the left at the winter solstice and to the right
at the summer solstice, i.e. exactly opposite to the elevations of the sun's path in order to cancel out the changes in inclination of the earth's axis and the inclination of the ecliptic caused thereby. At the equinoxes, the lemniscate axis would have to be exactly vertical, i.e. not performing a lateral inclination. Also, the position of the earth's axis would then be exactly in the middle between its two extremes of $35.25^{\circ}$ and $11.75^{\circ}$. That is, it would be $23.5^{\circ}$ at the equinoxes.


Figure 124: Lateral inclinations of the lemniscate axis

With this assumption, the situation at the time of the vernal equinox can now be drawn, as shown in Figure 125. The sun passes through the midpoint of the lemniscate. The earth reaches the left outer end of its path. If the earth's axis is to incline $23.5^{\circ}$ to the vertical and the sun is to cross the earth's path at the same time, the Earth must be raised to the height of a sun position, which the sun will not reach until half a year later, at the autumnal equinox.


Figure 125: The positions of the sun and the earth at the vernal equinox in the advancing lemniscate

Figure 126 shows the situation at the autumnal equinox. The sun again passes through the midpoint of the lemniscate. The earth reaches the right outer end of its path. To do so, it must descend to the height of the preceding position of the sun at the vernal equinox. The earth's axis inclines by $23.5^{\circ}$ and the sun can simultaneously cross the Earth's path.


Figure 126: The positions of the sun and the earth at the autumnal equinox in the advancing lemniscate


Figure 127: The upward movements of the sun and the earth in their advancing lemniscates

On the basis of the earth positions determined at the solstices and equinoxes, the upward movements of the earth can now be drawn in addition those of the sun (Figure 127). It can clearly be seen that the earth is running up or down at four different angles of inclination, while the sun follows its steady upward path. From the summer solstice to the autumnal equinox, the earth runs downwards at the same angle as the sun rises. This keeps the earth at the same height, i.e. it runs horizontally and still follows its lemniscatory path. Looking, not exactly from the side, but
obliquely from above into the advancing lemniscate paths of sun and earth, results in Figure 128. You can clearly see how the lemniscatory path of the earth winds around the more steady lemniscatory path of the sun. The strong ascent from bottom right to top left is also clearly visible, and the far smaller descent from top left to bottom right. The earth always goes three steps upwards (referring to the midpoints of the lemniscates) and then one step back again.


Figure 128: The advancing lemniscatory paths of the sun and earth with their vertical movements


Figure 129: The advancing lemniscatory paths of the sun and earth
Without the zigzag lines of the vertical movements, the two advancing lemniscatory paths with their complicated courses are even more clearly expressed (Figure 129). The common intersection points in the center of the lemniscates, mentioned by Rudolf Steiner, are retained. His following explanations, which he gave in connection with his sketch of the non-advancing double-lemniscate path of the sun and the earth (Figure 119, page 90), thus also apply to the situation in the advancing paths:
"But essentially, as you can see from this, there is a point in the universe where the sun and the earth are, only not at the same time, but approximately while the sun is there on its path (point [in Figure 119]), that is, has left this point by a quarter of its path, the earth begins with its movement in the point, which the sun has left. After all, we are really in space after a certain time at the place where the sun was; we follow in some way following the path of the sun, cross it, are at a certain time of the year where the sun was. Then the sun goes on [arrow to the right in Figure 119], the earth also [arrows to the top left in Figure 119], and after some time the earth is again approximately at the place where the sun was. We really go in space with the earth through the place where the sun was. We sail through it; but not only do we sail through, but the sun leaves consequences of its effect in the space it has to pass through, so that into the traces, into the remaining traces of the sun, the earth enters and crosses them, really crosses them. For space has living content, has spiritual content, and into what the sun effects, the earth enters and crosses it, sails through. ... Did people know of such things as this one here, that we enter the traces of the sun with the earth, that twice a year we are in there where the sun has worked in space? Yes, they actually knew about it, and it is even historically easy to prove that they knew about it. Just imagine, a person knows, knows correctly: at a certain time in the course of the year, the earth on its path crosses the sun's path in such a way that the earth enters the trace of the sun, in such a way that it follows the sun. The opposite is fulfilled when the earth returns to the other side. The one time it is as if the sun is descending under the earth's orbit, the other time as if the sun is rising and the earth's orbit is down. The one time it is as if the sun is descending under the earth's path, the other time as if the sun is rising and the earth's path is down. One time, man climbs so to speak over the sun's path with the earth, finds the trace of the sun, the other time he climbs down, climbs under the trace of the sun's path. - What could such a person say? Such a person could say: 'This is a particularly important moment for us; we are in the place where the sun has been!' And in the spiritual atmosphere this is expressed, because one encounters the image that the sun has left behind in the ether. There you set a feast! At this time you set a feast. - And two such feasts the old mysteries celebrated in the year, of which only remained - but, do not take it as if I wanted to state the right time - weak memories in today's feasts; but one no longer knows the connections. But in the old mysteries one knew: Now one crosses the sun's path in such a way that one finds sun content in the ether, which has been left. The fact that people have set up main feasts at certain times of the year has its guideline in such knowledge."

We may assume that these major festivals were the feasts of the winter solstice and the summer solstice, for just then the earth is cutting the path of the sun. Now, the solar globe has gigantic dimensions, i.e. the earth will certainly reach the "ethereally impregnated" path of the sun a few days before the exact winter solstice and will roam it for a few days afterwards. That's probably the reason why in the ancient mysteries the celebrations began as early as the middle of the month, as Rudolf Steiner continues to describe:
"With today's knowledge, people are separated from these connections. Today's people won't really respect things either, but they will say: 'Well, what do I buy myself for knowing that I'm in the same place where the sun was?' - That's what today's people would say. The ancient Egyptians, for example, did not say so in their mysteries. Because they knew: now the earth goes through the point, which the sun has once left, they asked, on the fifteenth of that month, that priestess, who was the priestess of Isis and was well prepared at hidden temple place, because they knew: through the special spiritual preparation that the priestess of Isis was able to undergo, the priestess of Isis brings forth what can be experienced, when one passes through the sun's aura. - And the priests have tried to hear from the statements of the priestess of Isis what she found in the sun's aura and wrote down: Rainy year, sowing the seeds at a certain time - in short, lots of practical things that were important for the guidance of the life in the next year. After that, one has followed well, because one knew how the sky had an effect on the earth. They tried to explore that. It was already decay time when this science was betrayed by the opponents of the Osiris-Isis ministry. One could only save oneself - this is the external event, which in turn is related to the single temple site in ancient Egypt, this art of walking the course of the year in this way and exploring the influences on the earth spiritually."

### 2.5.2 The effects of upward movements on the cosmic cross path of the sun and the earth

The movements of the sun and the earth were depicted in Figures 113 (page 82) and 116 (page 85) as a cosmic cross path. Later, a description of the two advancing lemniscatory paths followed. How can these two trajectories be linked? - To this end, it must be taken into account that the advancing lemniscatory paths shown in Figure 129 do not represent a rigid path system, but the upward movements depicted are supplemented by the rotation of the entire system and the pivoting motions of the lemniscates, which are sometimes even performed independently of each other. In spring only the Earth-lemniscate and in autumn only the Sun-lemniscate pivots by $60^{\circ}$ counterclockwise (see table page 83). In addition, the lemniscate axis tilts rhythmically in different directions, so that the earth's axis can always remain aligned with the celestial pole and the sun is not lifted out of the star constellations of the zodiac. The double-lemniscate of Figure 129 (page 96) behaves similar to a plant, which tends its longitudinal axis during the day in different directions and also follows the course of the sun with its leaves. Like plant leaves, the lemniscate halves "wag" around rhythmically over the course of a year and then return to their initial position. Thus, the advancing double lemniscate does not appear as a dead structure, but rather as a being permeated by an etheric body: a living being.

However, within a non-advancing lemniscatory path the cosmic cross path of sun and earth retains its validity. Looking down at it from the ecliptic pole, the two arms of the cross appear shorter than the longitudinal axis of the cross, because the earth's orbit (arms of the cross) is inclined to the sun's path (stem of the cross). This results in a perspective shortening of the Earth's path. Figure 130 shows the lateral inclination of the cross arms at $23.5^{\circ}$ in three dimensions. The lemniscate axis points in the direction of the ecliptic pole. At the winter solstice, the sun is far behind in the picture and is therefore drawn in small. It runs for half a year towards the viewer, to the summer solstice at the very front of the picture, therefore drawn in large scale. Then the sun follows the same route back until the winter solstice. The Earth's path runs at right angles to the sun's path and is inclined by $23.5^{\circ}$. The earth begins its course at the midpoint of the laterally inclined cross, running left up to the vernal equinox, then down to the right over the midpoint of the cross to the autumnal equinox, and then back to the midpoint of the cross.

The upward movements of the sun and the earth on their advancing lemniscatory paths naturally have an effect on the course of the cosmic cross path. The result is shown in Figure 131. The stem of the cross formed by the sun's path is "widened" to a zig-zag line (orange). The same happens with the cross arms (blue), while the inclination of the earth's orbit also comes into play. The inclination angles of the Earth's path correspond to those already shown in Figure 127 (page 95). Now, however, the sun's path is rotated by $90^{\circ}$ with respect to the Earth's path. The latter is as already mentioned the consequence of the pivoting and rotary motions of the advancing lemniscate.


Figure 130: The inclined cosmic cross path of Sun and Earth in the non-advancing double-lemniscate


Figure 131: The inclined cosmic cross path of Sun and Earth in the advancing double-lemniscate

The sharp bend in the course of earth's path each winter and summer solstice arises only within the reference system chosen here. It is ultimately annulled by the play of movements of lemniscate axis, which will now be described in more detail.

### 2.5.3 The lemniscatory motion of the lemniscate axis

In the context of the considerations on the position of the earth's axis in an advancing doublelemniscate, the necessity of rhythmic lateral inclinations of the lemniscate axis has already been pointed out (Figure 124, page 94). If the sun's path rises to the left, the lemniscate axis must incline to the right. If the sun's orbit rises to the right, the lemniscate axis must incline to the left. In this way, all elevations of the sun's path are compensated by an exactly opposite inclination of the lemniscate axis. Rudolf Steiner, however, does not speak of simple lateral inclinations, but of the "axis itself becomes a lemniscate again" (Lecture of 2 May 1920 [5]).

But why should a lemniscatory motion of the axis be required, which must contain additional forward and backward inclinations, instead of the simple lateral inclinations? The reason lies in the advancing of the lemniscate. An advancing lemniscatory path is not a rigid structure. It is in an ongoing formative process. The resulting consequences for the inclinations of the paths of the sun and the earth can be better observed by choosing a time point which lies exactly in the middle between a solstice and an equinox. This is the case e.g. on 4 February. In Figure 132, at the top left, the positions of the sun and the earth are initially represented in a non-advancing lemniscate. For a closer examination of the formative process of an advancing lemniscate it is useful to rotate this sketch by $90^{\circ}$. This is shown in the same figure at the top right, with only the front lemniscate halves showing, in which Sun and Earth are located during this period, for better clarity. The earth's axis inclines with its upper part away from the observer into the spatial depth, while the lower part of the earth's axis points towards the viewer.


Figure 132: Formative process of the advancing lemniscate

Figure 132 shows in the lower half of the drawing as by slow and even elevation of the sun's path (orange dashed) in the left part of the drawing the advancing lemniscatory path arises (red dotted arc line). The elevation results in a corresponding lowering of the sun's and earth's paths on the opposite side. Sun and earth are drawn red at their resulting new positions. The earth's path sinks downwards by $11.75^{\circ}$ and the earth's axis perpendicular to it tilts to the right by the same angle. Both can only be corrected by tilting the lemniscate axis, which had to incline $11.75^{\circ}$ to the right for the path elevation, by the same angle to the left. This now also raises the earth's path "progressively" and the astronomical laws are respected again. On 4 February of a year, the lemniscate axis must therefore complete a further inclination of $11.75^{\circ}$ at right angles in addition to the lateral inclinations described above. Overall, the axis movements required to form an advancing lemniscate can be summarized as shown in Figure 133. Ultimately, this results in a lemniscatory axis motion, exactly as Rudolf Steiner described. In addition to his statements, the exact angles of inclination of the lemniscate axis can now be specified during the course of the year. All in all, the axis inclination from left to right as well as from front to back covers the wellknown $23.5^{\circ}$.


Figure 133: Lemniscatory axis motion of the advancing double-lemniscate

### 2.6 The sun's movement towards the solar apex

Around the turn of the century from the 18th to the 19th century, Friedrich Wilhelm Herschel observed a movement of the sun towards the constellation of Hercules. The target point of the solar path he called Apex (Latin: peak, summit). In the course of the following two centuries, various attempts were made to determine the exact position of the solar apex astronomically and mathematically, which turned out to be a rather difficult task. Today it is assumed that the target point of the sun's path is close to $\mu$ Herculis, a less bright star of the constellation Hercules, not far from the brightly shining Vega in the constellation Lyre, and at about $30^{\circ}$ declination above the celestial equator. Projected onto the plane of the zodiac, the calculated solar apex lies almost exactly over the ecliptical length $0^{\circ}$ Capricorn (Figure 134).


Figure 134: Solar apex and orientation of the Sun-Earth lemniscates in space
A movement of the sun towards the apex can only occur if the double-lemniscate of the sun and the earth not only advances "vertically" in the direction of the ecliptic pole, but in addition is drawn "horizontally" in the direction of $0^{\circ}$ Capricorn, the point of the winter solstice. The tractive forces are likely to emanate from the outer planets. The resultant of both movements would then be the apex movement of the sun (red arrow in Figure 134). In fact, Rudolf Steiner drew a sketch for such a "horizontal movement" of the Earth-Sun lemniscate in the lecture of 17 January 1921 [3] and called this type of movement "lemniscatory helical line" (Figure 135). He has sketched here a "lateral advancing" of the lemniscate.


Figure 135: Rudolf Steiner's sketch on the lateral advancing of the Earth-Sun lemniscate [3]

Of course, this is also associated with an upward movement, as we know from other sketches of Rudolf Steiner, which have already been looked at in more detail. However, such a movement is not necessarily evident from the given two-dimensional sketch and can also be neglected for the following considerations initially. The added large circle represents the apparent orbit of the Sun ( S and S 1 ) in the sky observed by the respective position of the Earth (E and E1). Earth and Sun
run on a common lemniscatory path, which advances from left to right. Rudolf Steiner was only able to draw the positions of the sun and their projection on the circle in one of the five sketched halves of the lemniscate, because on a lemniscatory path the sun can only temporarily maintain the direction of motion drawn in counterclockwise direction (from $S$ to $S 1$ ). The possibility of being able to project the sun, seen from the earth, in a constant direction of motion onto the drawn-in large circular path ends with the transition of the sun to the next lemniscate half, because in doing so the sun must perform a change of direction and move exactly the opposite way for some time, i.e. clockwise. Consequently, Rudolf Steiner did not draw any earth and sun positions in the following, downward facing lemniscate half. The maintenance of an apparent circular path of the sun is only possible if one subjects the lemniscate, which is to be thought of as consisting of two lemniscates *, all the pivoting and rotating motions described above in detail, which are sometimes even performed separately (Figures 100 to 112, starting on page 75). Naturally, Rudolf Steiner was unable to depict these complicated movements in a single, motionless sketch.

The sketched steady lateral advancing of the lemniscate from left to right can only be understood as a movement of the sun in the direction of $0^{\circ}$ Capricorn. Contrary to Figure 134, where the self-contained double-lemniscate in the center of the globe points towards $0^{\circ}$ Capricorn with its longitudinal axis, in the sketch of Rudolf Steiner the longitudinal axis of the lemniscate is at right angles to the direction of its advancing.

The lateral advancing is by no means in contradiction to the already described pivotal lemniscates, but can be reconstructed with them. At the times of the vernal and autumnal equinoxes, the Earth-Sun lemniscate is always rotated such that its longitudinal axis is at right angles to the direction of the apex or towards $0^{\circ}$ Capricorn. From Figures 103 (page 77) and 109 (page 80), the corresponding positions of the lemniscates can be easily adopted together with the positions of the sun and the earth. Since in these pictures $0^{\circ}$ Capricorn is at the top in the middle of the picture, the Lemniscates must be turned $90^{\circ}$ clockwise to correspond to the alignment in Rudolf Steiner's sketch. Now they can be moved laterally at regular intervals. In Figure 136 on the far left, the situation at the autumnal equinox is drawn in brighter colours.

After half a year, at the time of the vernal equinox (darker colors), the lemniscate has moved a bit to the right in the direction of $0^{\circ}$ Capricorn. Thus, vernal and autumnal equinoxes alternate in halfyearly steps, always standing at right angles to the direction of movement. The yellow bar running across behind the sun's positions represents the straight-linear solar path, i.e. the stem of the cross within the cosmic cross path of the sun and the earth. At right angles to this, the earth carries out its straight-linear motions on the cross arms, with earth and sun being pulled together in the direction of $0^{\circ}$ Capricorn.


Figure 136: The lateral advancing of the Earth-Sun lemniscate in half-yearly steps
in the sense of the double-lemniscate path of the sun and the earth

The entire sequence of movements can be simplified depicted by combining the half-year steps with a connecting line (black) (Figure 137). Since the Lemniscate sets off immediately after the autumnal equinox, but also reaches its next temporary final position only with the next vernal equinox, the loops must inevitably be drawn slightly narrower than the lemniscate-halves.


Figure 137: The connecting line to the lateral advancing of the Earth-Sun lemniscate
The black connecting line can now be compared with the sketch of Rudolf Steiner. The great similarity of both lines confirms the concordance of his sketch with the above considerations on the pivotal lemniscates and the cosmic cross path of the sun and the earth, although Rudolf Steiner's sketch only reproduces a small, quite fundamental excerpt from the complicated structure of motion (Figure 138). But all his sketches have this in common. They always show the most important basic principles.


Figure 138: Comparison of the connecting line to the lateral advancing of the Earth-Sun lemniscate with Rudolf Steiner's sketch

The narrowing of the lemniscate halves can be more accurately comprehended by letting the lemniscate, consisting of two semicircles, as shown repeatedly in Figures 136 and 137, advance monthly by one-twelfth of the distance that the sun moves in the direction of $0^{\circ}$ Capricorn over the course of a year. The positions shown in red in Figure 139 refer to the earth. However, they are not
real earth positions, because here the rotary and pivoting motion of the lemniscate is not taken into account. Real positions of the earth are only the three light blue marked positions at the time of vernal and autumnal equinoxes. Nevertheless, in order to describe the principle of the advancing of the lemniscate towards the solar apex, one can proceed as Rudolf Steiner did, by establishing a connection between those positions of the lemniscates, which are always aligned again the same way after half a year, i.e. with the longitudinal axis at right angles to the movement towards the solar apex.


Figure 139: Graphical determination of the connecting line to the lateral advancing of the Earth-Sun lemniscate

If the narrower lemniscate halves determined in this way are detached and then combined to a self-contained lemniscate (both steps are illustrated in the right half of Figure 139), the result is exactly the form of a lemniscate that Rudolf Steiner sketched in his lecture of 1 October 1916 [1] as the basic form of the Earth-Sun lemniscate. In Figure 140, both forms of a lemniscate are juxtaposed. The newly determined one was turned by $90^{\circ}$. Now it can finally be explained why on the one hand almost circular lemniscate halves are absolutely necessary in order to maintain an even distance between the earth and the sun within a lemniscate half, on the other hand the sequence of movements can be described just as accurately with the narrow lemniscate sketched by Rudolf Steiner.


Figure 140: Comparison of the determined lemniscate connecting line (top) with Rudolf Steiner's sketch to the basic principle of the lemniscatory path (bottom) [1]

If you want to draw the real path of the earth, as it results from a rotating and horizontally advancing pivotal lemniscate, you must make the lemniscate's midpoint move in twelve monthly steps towards $0^{\circ}$ Capricorn and enter the earth's positions. A second method, which leads to the
same result, is to allow the cosmic cross path of the sun and the earth, rotated by $90^{\circ}$, to take twelve monthly steps. Both methods lead to a looping movement of the earth around the straightlinear motion of the sun (along the yellow cross stem), as Figure 141 shows.


Figure 141: The earth's path with lateral advancing of a rotating pivotal lemniscate

A similar path of the earth in connection with the movement of the sun towards the apex was sketched by Rudolf Steiner on 29 April 1908 [10] already and described in the following words: "In reality, the sun races at high speed through space towards the constellation Hercules. Such a movement, as it is usually described, is faked only by the fact that the planets move along with it. The true path of the earth forms a helix. What is called the obliquity of the ecliptic is the line of gravity between the sun and the earth. ..." Rudolf Steiner describes the earth's path as a "helix". However, we must take into account that in the loopy motion determined above, the additional, upward advancing of the lemniscate, which raises the process to three-dimensional, is not included. A two-dimensional view therefore results in a loop line. Figure 142 compares the loop line of the Earth's path with the sketch Rudolf Steiner's sketch. The principle similarity of both pictures is striking.


Figure 142: Comparison of the graphically determined motion of the earth and the sun with Rudolf Steiner's sketch to the apex motion of the earth and the sun

All in all, however, the vertical and horizontal advancing of the Earth-Sun lemniscate described by Rudolf Steiner is not sufficient to explain the declination of the solar apex of $30^{\circ}$ above the celestial equator or $53.5^{\circ}$ above $0^{\circ}$ Capricorn on the ecliptic. The raising path of the sun with an inclination angle of $23.5^{\circ}$ (Figure 118, page 91) results in an annual vertical movement of approx. 0.87 AU . From Rudolf Steiner's sketch on the lateral advancing of the Earth-Sun lemniscate (Figure 135, page 102) an annual horizontal movement of approx. 1.4 AU can be deduced. The resultant of both movements points to a point $30^{\circ}$ above the $0^{\circ}$ Capricorn point on the ecliptic. This corresponds to a declination of $6.5^{\circ}$. There is still a missing $33.5^{\circ}$ to the calculated apex position of about $30^{\circ}$ declination. Ultimately, this means that there must be additional forces that pull the Earth-Sun lemniscate towards the solar apex. We can assume the origin of these forces in the outer planets, especially in the planet Saturn, of which Rudolf Steiner says: "... if we take this outermost planet of our solar system, Saturn, then we have to imagine it ... as the leader of our planetary system in world space. It pulls our planetary system in the world space. It is the body of the outermost force that guides us around in the lemniscate in the world space. It drives and pulls at the same time. So it is the force of the outermost periphery."(Lecture of 2 May 1920 [5]) Two effects of force apparently go out from Saturn: First, it causes the sun and earth to run on lemniscatory paths, and on the other hand, it also draws the entire system through the space towards the solar apex. The resulting path has its own angle of inclination, as can be seen from the sketch Rudolf Steiner painted on the board for his remarks at the conference of 25 September 1919 [4] (Figure 143). He adds about the angle of inclination: "The earth follows the sun. The slope is equal to what is called the declination angle; if you take the angle you get out of it, if you take the ecliptic angle that it encloses with the equator, you'll get that out." Accordingly, $23.5^{\circ}$ would have to be added to the above-mentioned $6.5^{\circ}$ declination resulting from the vertical and horizontal advancing of the Earth-Sun lemniscate, resulting in a declination of $30^{\circ}$ as calculated also by modern astronomers.


Figure 143: Rudolf Steiner's sketch for his remarks at the conference on 25 September 1919 [4]

## PART 3

Summary: The multi-part series of considerations on the statements of Rudolf Steiner concerning lemniscatory, circular, straight and helical paths of the planets ultimately leads to the realization that we are dealing here with altogether five formative steps of complicated path forming processes. The middle of the five formative steps is the cosmic cross path of the sun and the earth. Twelve cosmic path forming forces are involved in the design of these formative steps. The results of their efficacy, the ultimate outer forms, are the well-known Copernican planetary orbits. Rudolf Steiner's concern was evidently, to lead astronomy from the consideration of external forms to a consideration of their underlying formative processes and thus to develop the physical thinking from pure form-thinking to a moved thinking of formative forces. In the twelve formative forces, which can be attributed to the twelve zodiac signs, and the cosmic cross path of the sun and the earth as the thirteenth in their midst reveals the efficacy of Christ's being also in the field of the path forming of the planets.

### 3.1 Introduction

The sciences, originally born from the mystery schools of antiquity, have undergone increasing materialization and mechanization over the centuries. An understanding of the psycho-spiritual and the living as independent modes of being has been completely lost. Today, science assumes that from a certain complexity and diversity of inanimate matter, life simply comes into being. An even more extensive complexity and diversity should then even produce the psycho-spiritual. Accordingly, life is regarded as a subsequent phenomenon of the inanimate, of the dead, and consciousness as a subsequent phenomenon of life. With this basic philosophy, modern science takes just the reverse view than that of the mystery schools of antiquity. At that time it was taught that the creation originated from a purely psycho-spiritual, which brought forth the life out of itself, which in a further, later step secreted the inanimate and still secrets today as in the generation of the plants' trunks and barks, the shell generation of the mussels, the scaling of the fishes, the feather generation of the birds, the generation of hairs, claws or solidified hooves in the higher animals, or the generation of hairs, nails and the dying, flaking, outermost layer of the human skin. The outermost sheaths of living beings and all forming processes in nature were explained as final products, as the last result of the interaction of various life processes and not vice versa. In contrast, modern science regards the dead matter visible to the physical senses as the starting point of all development. With regard to its numerous physical observation techniques, science has even reached a certain climax. At the same time, however, it has reached a low point by declaring the dead substance the origin of life and consciousness. With this, mankind has finally completely stepped out into the physical-mineral, dead world of forms. But now it is called to overcome death, to turn its view to life again, to find its way knowledgably into the interrelationships of formative forces and thereby to revive and spiritualize the sciences step by step.

With JUPITER Vol. 5, No. 1, September 2010, a series of considerations has been started, which are intended to provide a contribution in leading astronomy as one of the numerous branches of science, by taking up and evolution of Rudolf Steiner's numerous suggestions on the planetary movement, a first step towards an understanding of living form creating processes. At the center of the considerations are not the bodies of the planets, but their paths and their processes of creation through the interaction of different types of cosmic formative forces. It remains undisputed that modern astronomy is able to calculate the positions of the planets, seen from the
earth, quite accurately ahead and back in time. However, Rudolf Steiner points out that the real motion of the planets in space is a completely different one and that only as a result of complicated path processes those planetary positions emerge, as they can also be described in Copernican terms and grouped into circular orbits or ellipses. But, the principle of the circle or ellipse, which has hitherto been recognized in the history of astronomy, is only one of three. Rudolf Steiner adds the lemniscate and the straight-line as further principles of planetary motion and path formation. Circle, lemniscate and straight line are not only the form principles of the macrocosm, but also of the microcosm, the human being. According to the principle of the circle or the ellipse the human head is constructed, according to the principle of lemniscate the circulatory system of man and according to the principle of the straight line the human limbs are designed. The human body and the paths of the planets are thus likewise images of cosmic-etheric formative forces. Figure 144 shows the evolutionary steps taken so far by astronomy and how the next step leads to the formative forces of the planetary paths.


Figure 144: The evolutionary steps of astronomy
To trace these formative forces of the planetary paths, to understand the different steps of the formative processes and to learn to understand how ultimately the astronomically observable forms of the planetary orbits arise, is the concern of the series of considerations on the lemniscatory path system. In the present PART 3 the picture is now rounded off and an overview becomes possible of the twelve formative forces effective in our planetary system, which over five formative steps ultimately lead to those path forms, as they can also be described in the Copernican way.

### 3.2 The lemniscatory paths of the inner planets

At the beginning of the considerations on the lemniscatory path system, the question was raised as to whether the paths of the inner planets observable in the sky, with their exceedingly variable loops, peaks and curves, could be an expression of a lemniscatory movement of the sun and the earth, as described by Rudolf Steiner. It could be shown that the leading of Mercury and Venus along an upwardly advancing double-lemniscate path of the sun and the earth yields those conspicuously shaped path sections of the inner planets in comparable periods of time, which are produced by the Copernican contemplation as well. The resulting paths of Mercury and Venus show recurring changes in direction and thus the main feature of a lemniscatory path. From this point of view, the graphically determined paths could be called Mercury's and Venus' lemniscate.

But actually both planets maintained their Copernican orbit with respect to the sun so as not to violate today's recognized astronomical laws. The question therefore arises as to whether Rudolf Steiner did mean something else when he spoke of the lemniscatory paths of the inner planets. However, what other possibility could there be for a lemniscatory motion of Mercury and Venus that can be reconciled with their elliptical orbits?

If one draws in the sketch of Rudolf Steiner from the third scientific course (lecture of 17 January 1921, Fig. 6 [3]) the Copernican orbits of Mercury and Venus, Figure 145 emerges. One can clearly see how small the sketched lemniscatory orbits of Mercury and Venus are in comparison to their Copernican trajectories around the Sun. Even if the small lemniscatory paths were rotating or pivoting back and forth, Mercury and Venus could never take those positions to the sun as they are proven today. Also, Rudolf Steiner's sketch is not quite to scale in contrast to some other of his sketches. Did he perhaps not want to reproduce the real lemniscatory paths here, but only the principle of their connection to the double-lemniscate of the sun and the earth? In this case, he did not need to attach any importance to a true-to-scale sketch. It was then only important to depict the connection of the lemniscate midpoints of the inner planetary paths with the EarthSun lemniscate and to maintain the Copernican planetary order Earth, Venus, Mercury and the Sun.


Figure 145: The Copernican orbits of Mercury and Venus drawn in Rudolf Steiner's sketch

From the remarks of Rudolf Steiner to his sketch it also follows that not the real lemniscatory paths were drawn, but that he was obviously only concerned with the representation of the principle of their connection to the Earth-Sun path: "Of course, the lines that I have drawn here are only meant schematically, and actually one would have to say: an inner planet has a path that makes a loop whose center is the Earth-Sun path itself; an outer planet picks up the Earth-Sun path in its loop. - That is what is really essential, because the matter itself is so extraordinarily complicated that you can actually only get to the schematic ideas."

In addition, Rudolf Steiner points out, at least with regard to the outer lemniscates halves, these are much larger than those sketched by him and run "seemingly to infinity": "If you now think of this here as the Earth-Sun path, then you are forced in order to bring out in perspective the forms that the other planetary paths have with the course of the sun, to imagine the paths of the planets
close to the sun in such a way, that they could be drawn in like this. This gives you the possibility, if you have the sighting line here, of finding the loop as a perspective structure in a certain other position of the planet on the path. The sighting line (v) is here. We get the loop (s) here and these two branches [the two outer lemniscate halves] seemingly run to infinity (u)."

This raises the question: is it purely geometrically possible that Mercury and Venus run on lemniscatory paths, which would have to be much larger than those sketched, but are nevertheless bound to the "Earth-Sun path" in the illustrated manner, and which by motion, e.g. rotating or pivoting, lead to the result that both planets seem to run on circular or elliptical orbits around the sun? In PART 1 of the considerations, the similarity of an elliptical orbit with a lemniscate half has already been pointed out. So, another question arises: Do the Copernican orbits of the inner planets perhaps simply correspond to a lemniscate half of their complete lemniscatory paths?

In the following we will try to solve both questions graphically. For this purpose it makes sense to use a two-dimensional, non-advancing "Earth-Sun path" as a basis. As the observations of the preceding sections have shown, this is by no means a rigid path system, but the earth's and the sun's lemniscates perform completely at times independent turns and still remain connected to each other in their lemniscate midpoints. In the same way, by the lemniscate midpoint, according to Rudolf Steiner, the orbits of the inner planets are also connected to this path system, whereby we can interpret another sketch from a notice sheet of Rudolf Steiner (Figure 146 - sketch from notice sheet 121) to the effect that the lemniscate midpoints of the paths of the inner planets (drawn as a representative small lemniscate) are only connected to the sun's lemniscate and not to the earth's lemniscate, from which the sun's occasionally turns away. This also corresponds to the Copernican view that the planetary bodies orientate themselves in their course to the sun and not to the earth.


Figure 146: Rudolf Steiner's sketch from notice sheet 121

All considerations on the lemniscatory path system in PART 1 and PART 2 are based on another sketch of Rudolf Steiner from the lecture of 1 October 1916, in which he gives the principle of the double-lemniscate path of the sun and the earth (Figure 147). It can be interpreted as a constellation at the time of the winter solstice (see Figure 148). In the sketch, the sun moves clockwise. This process of movement was consistently followed up. For this purpose, in all previous drawings, the zodiac also had to be mapped in the direction of the given course of the sun, i.e. clockwise running. In PART 2 the "ecliptical pole" was indicated as the position of the observer in the figures with a view vertically from above down to the zodiacal plane. If one had
any reason for the clockwise order of the zodiac signs to be given, it should have called "the ecliptic South Pole". But it is unlikely that Rudolf Steiner, as a resident of the northern hemisphere, drew his sketches with a view from the ecliptic South Pole. Rather, it is to be assumed that he drew imaginatively perceived images from the astral world, in which everything appears reversed, as a sketch on the blackboard.


Figure 147:
Rudolf Steiner's sketch of the double-lemniscate path of the sun and the earth


Figure 148:
Interpretation of Rudolf Steiner's sketch as constellation of the winter solstice

Rudolf Steiner's side-inverted view had to be maintained until the resulting consequences could be understood and graphically reconstructed. Only now is it possible to break away from it and transfer everything to the directions valid for the physical world. With the view from the ecliptical North Pole or the Sky North Pole, which is common in astronomy, the zodiac runs counter-clockwise. This way of presentation should therefore be used in all following figures.

Likewise, the movements of the sun and the earth on their lemniscatory paths, sketched by Rudolf Steiner, must be reversed in their direction. The pivoting motions of the sun's and the earth's lemniscates described in the table on page 83 must be changed from $-60^{\circ}$ to $+60^{\circ}$. The year-round continuous rotary motion must also be changed from $+30^{\circ}$ per month to $-30^{\circ}$ per month (zodiac month):

| Winter | Spring | Summer | Autumn |
| :---: | :---: | :---: | :---: |
| Common <br> pivoting motion <br> of both lemniscates <br> by $+60^{\circ}$ | Sole <br> pivoting motion of <br> the earth's lemniscate <br> by $+60^{\circ}$ | Common resting <br> of both lemniscates | Sole <br> pivoting motion of <br> the sun's lemniscate <br> by $+60^{\circ}$ |
| Year-round continuous rotary motion of $-30^{\circ}$ per month (zodiacal month) |  |  |  |

The term "pivoting motion" was chosen at a time when the need for the temporarily separate motions of the sun's and the earth's lemniscates was not yet recognized. As described in PART 1 of these considerations, the course of the sun on a common lemniscatory path of sun and earth can only form a circular path, if this pivots for half a year to the left and then for half a year to the
right. This movement pattern was called "pivoting motion". PART 2 describes the necessity of temporarily separate motions of the sun's and the earth's lemniscates. However, these always take place in the same direction. The term "pivoting motion" loses its meaning. The following table therefore indicates a half-year "rotation" of $+60^{\circ}$ per month (zodiac month) both for the earth's lemniscate as well as for the sun's lemniscate. Each of the two lemniscates rotates $6 \times 60^{\circ}$, i.e. a full $360^{\circ}$, over the course of half a year. Instead of a pivoting motion, they both perform a complete rotation around their lemniscate midpoint within half a year and then rest for another six months. In order to be able to show the rotation times in a coherent way, the following table starts with autumn.

| Autumn | Winter | Spring | Summer |
| :---: | :---: | :---: | :---: |
| Rotation of the sun's lemniscate <br> by $+60^{\circ}$ per month | Resting of the <br> sun's lemniscate | Common resting of <br> both lemniscates |  |
| Resting of the <br> earth's lemniscate | Rotation of the earth's lemniscates <br> by $+60^{\circ}$ per month |  |  |
| Year-round continuous rotation of the double-lemniscate system |  |  |  |
| by $-30^{\circ}$ per month |  |  |  |

With regard to the lemniscatory paths of the other planets, the following questions arise: Are the laws of the lemniscatory paths of the sun and the earth found and described in the table also valid for the lemniscatory paths of the other planets? Are they transferable and maybe just need to be modified a bit? Or are the lemniscatory paths of the other planets completely subject to their own laws? The following considerations try to provide solutions to these questions.

### 3.2.1 The Lemniscate of Mercury

The first step in solving the above mentioned questions is to gain clarity on how the orbit of Mercury, which is to be considered in more detail, is oriented in the zodiac. The Copernican orbit of Mercury is the most elliptical of all the orbits of the seven traditional planets. According to Kepler, the sun is located in one of the two focal points of the ellipse. If you draw a straight line from the point furthest from the sun (aphelion) of Mercury's orbit to the point nearest to the sun (perihelion) - the apsidal line - in 2011 it points, seen from the sun, to about $18^{\circ} \mathrm{Gemini}$ (Figure 149). These data are verified by observations and cannot be seriously questioned today. However, one can well ask oneself whether the orbit calculable as an ellipse describes the original impulse of Mercury's motion, or whether it only appears as a result of different sequences of motion, e.g. also of lemniscatory motion. Does Mercury perhaps not run on an elliptical orbit at all, but on a lemniscatory path as Rudolf Steiner describes it? Is this possibly subject to the same laws as were found and described above for the lemniscatory paths of the sun and the earth? Does Mercury's lemniscate also rotate $60^{\circ}$ per month and for half a year once around its own axis? Does it then also rest for another half of a year? Is it in addition subject to a year-round continuous rotation like the double-lemniscatory system of the sun and the earth?


Figure 149: Orientation of Mercury's orbit in the zodiac

Already with the very first considerations on the lemniscatory path system the question was raised as to whether the Copernican elliptical orbits perhaps simply correspond to lemniscate halves. For the elliptical orbit of the earth this has not been confirmed. The Copernican earth orbit has the same longitudinal diameter as its lemniscatory path, i.e. the sum of the longitudinal axes of its two lemniscate halves. But, does this have to be the case with the other planets also? After all, the double-lemniscate system of the sun and the earth seems to be a peculiarity.

Figure 150 shows a suspected Mercury lemniscate. There you can see its size relative to Earth's orbit. At the time shown the upper (thinner) lemniscate half of Mercury's path corresponds to the Copernican orbit of Mercury. Its perihelion is the lemniscate midpoint. Mercury was there, for example, on 31 October 2007. Seen from the earth, it passed its perihelion at that time in $24^{\circ}$ Libra. Mercury needs 88 days to complete its elliptical orbit. This period corresponds to its "elliptic year". For a full round on the marked lemniscatory path it would need twice the time or 176 days. A "lemniscatory year" thus corresponds to two "elliptic years". A "lemniscatory month" for Mercury would therefore be one twelfth of its "lemniscatory year" of 176 days, i.e. about 15 days ( 14.7 days). Figures 150 to 156 show the trajectory of Mercury, if its lemniscate is allowed to rotate in accordance with the laws of the Earth-Sun lemniscate. However, the lemniscates of Mercury must rotate in the opposite direction. Its rotation angle is therefore not $+60^{\circ}$, but $-60^{\circ}$. In addition, the rotation angle is not constant here. In order to meet the strong elliptical shape of the Mercury orbit, it varies by $\pm 10^{\circ}$. The exact angles of rotation are initially decreasing - $70^{\circ},-60^{\circ}$, $-50^{\circ}$ and then rising again $-50^{\circ},-60^{\circ},-70^{\circ}$ over a period of six lemniscatory months. All in all, in half a lemniscatory year, i.e. in 88 days, the lemniscate of Mercury performs a full rotation of $360^{\circ}$.

Mercury runs clockwise on its rotating lemniscatory path in figures $150-156$, but nonetheless seems to run counter-clockwise on an elliptical orbit at the same time. The latter is the outward appearance that results from the interaction of planetary lemniscatory movement and path rotation. The above-mentioned angles of rotation are also confirmed by halving the movement steps of Mercury to 7.5 days for control. Then, its lemniscatory path turns three times $-35^{\circ}$, then six times
$-25^{\circ}$ and finally again three times $-35^{\circ}$. The result is: $\left(-35^{\circ}\right)+\left(-35^{\circ}\right)=-70^{\circ}$ and $\left(-35^{\circ}\right)+\left(-25^{\circ}\right)$ $=-60^{\circ}$ and $\left(-25^{\circ}\right)+\left(-25^{\circ}\right)=-50^{\circ}$ and then the reverse, ascending order: $-50^{\circ},-60^{\circ}$ and $-70^{\circ}$.


Figure 150: Position of Mercury's lemniscate on 31 October 2007


Figure 151: Position of Mercury's lemniscate on 15 November 2007


Figure 152: Position of Mercury's lemniscate on 30 November 2007


Figure 153: Position of Mercury's lemniscate on 15 December 2007


Figure 154: Position of Mercury's lemniscate on 29 December 2007


Figure 155: Position of Mercury's lemniscate on 13 January 2008


Figure 156: Position of Mercury's lemniscate on 28 January 2008

Mercury's positions are all on the Copernican elliptic orbit. If Mercury's path were not an ellipse but a circular path, its lemniscate could turn at a constant - $60^{\circ}$. This suggests that the variability of the other planetary lemniscates is less than $\pm 10^{\circ}$ and thus remains closer to the mean of $-60^{\circ}$.

On 28 January 2008, Mercury again reaches its perihelion in the lemniscate midpoint (Figure 156). From now on, Mercury's lemniscate rests for half a lemniscatory year. This corresponds to the laws that have already been described for the Earth-Sun lemniscate. Mercury now continues to run counterclockwise according to the change of direction in a lemniscate. During this period, the lemniscatory path and the Copernican elliptical orbit coincide completely. Only after 88 days have passed, when Mercury reaches again its perihelion in the lemniscate midpoint, Mercury's lemniscate will rotate again for half a year.

Obviously, Rudolf Steiner was not interested in explaining away the Copernican orbits, but in pointing out that one cannot deduce the impulse of motion inherent in the planet from the final form of the planetary path. This is lemniscatory or subject to a lemniscatory force. The final path, the outer appearance of the Copernican elliptical orbit, is created only by the interaction of various additional path-forming forces.

The lemniscatory paths of the planets apparently show three polarities. They were already readable on the earth's and the sun's lemniscate, but only now, by the lemniscate of Mercury, they find their confirmation for the other lemniscatory paths also.

1. The two opposite and symmetrically mirrored lemniscate halves show a "polarity of form".
2. The opposite direction of the planets' track in the two lemniscate halves (clockwise and counterclockwise) corresponds to a "polarity of direction".
3. Half-yearly rotation and half-yearly resting can be described as "polarity of behavior".

At the realization of the outer appearance of the Copernican elliptical orbit only the lemniscate half on which the planet runs for half a year is always "actively" involved. The other lemniscate half points away from the sun during this time, out into space. It does not appear further. Presumably Rudolf Steiner meant this circumstance when he said about the outer loops or "branches" of the lemniscatory paths of the inner planets sketched by him: "... these two branches seemingly run to infinity (u)." (cf. p. 112)

In part, the laws of Mercury's lemniscate are very similar to those of the earth's lemniscate and the sun's lemniscate. In both cases, we find a half-yearly rotating and resting, as well as a rotation angle of $60^{\circ}$ (with a maximum variability for the extremely elliptic Mercury orbit, of $\pm 10^{\circ}$ ). The degree of rotation is at least temporarily the same. But, the direction of rotation is exactly the opposite. Completely different are the sequences of movement with respect to another regularity observed at the Earth-Sun lemniscate. Mercury's lemniscate - just as the lemniscate of Venus, as will be seen - is not subject to the year-round continuous rotation of $-30^{\circ}$ per month. The latter is a peculiarity of the double-lemniscate system of sun and earth. Therefore, it is no longer possible to speak of a system rotation of the inner solar system, but only of a system rotation of the doublelemniscate system Earth-Sun.

The connection of the lemniscatory path of an inner planet to the Earth-Sun lemniscate is apparently made via the perihelial point of its orbit. It is the pivot around which the lemniscate rotates: the lemniscate midpoint. Most of the time, however, it is not on the Earth-Sun lemniscate, as this is constantly changing its position. Even if no half-yearly rotation is present, it is subject to the year-round continuous rotation. Thus, only occasionally in the course of a solar year, a contact can occur between Earth-Sun lemniscate and the perihelial point or lemniscates midpoint of an inner planet. In this respect, the time should be important for which Rudolf Steiner has sketched the planetary positions. He set the sun in its own lemniscate midpoint. This is the case only at the equinoxes. So it makes sense, to check whether the Earth-Sun path cuts off the midpoint of Mercury's lemniscate - the Mercury perihelion - at one of the equinoxes, or perhaps both. As shown in Figures 157 and 158, the Earth-Sun lemniscate comes extremely close to the perihelion or lemniscate midpoint of Mercury's path at the equinoxes. Mercury's lemniscate itself is not drawn because it does not follow the rhythm of the solar year and is in different positions at the equinoxes of the successive years. It is understandable, however, from the two illustrations, why in his sketch Rudolf Steiner put the midpoint of Mercury's lemniscate on the Earth-Sun lemniscate. Also it should be noted that the perihelion or the apsidal line of Mercury's path is subject to the socalled perihelial rotation or apsidal precession. The perihelial point of Mercury's orbit is not fixed but moves clockwise around the sun at 5.74" (arc seconds) per year. For a full circulation of Mercury's perihelion this results in a period of approximately 225,000 years. Understandably no one has been able to observe such a calculated full circulation. Perhaps the perihelion or lemniscate midpoint only swings a bit back and forth and remains in the nearer surroundings of the position of the sun's lemniscate at the equinoxes.

A direct contact of the lemniscate midpoint of the Mercury's path with the sun's lemniscate could also come about, if one imagined the path of the sun as broad as the sun itself. On another occasion, reference has already been made to Rudolf Steiner's words, according to which the sun ethereally impregnates the space it passes through, that is to say its path. Thus, its lemniscatory path should have a width corresponding to the sun itself, or even be wider, if one imagines the etheric effect going beyond the physically visible sun. Then the lemniscate midpoint of Mercury's path would actually be on the sun's lemniscate at the equinoxes.


Figure 157: Position of the perihelion and the lemniscate midpoint of Mercury's path at the vernal equinox


Figure 158: Position of the perihelion and the lemniscate midpoint of Mercury's path at the autumnal equinox
"We really go in space with the earth through the place where the sun was. We sail through it; but not only do we sail through, but the sun leaves consequences of its effect in the space it has to pass through, so that into the traces, into the remaining traces of the sun, the earth enters and crosses them, really crosses them. For space has living content, has spiritual content, and into what the sun effects, the earth enters and crosses it, sails through. ... There is also such a connection with the other planets. We are at certain times in the places where Mercury was and so on. Very complicated movements are carried out by the planets in space, in outer space, but they enter into the traces of each other. Now you have the external image, the purely geometric external image [notice: the Copernican ellipse orbit]; the other image will be added, and it is only from the unification of the two images that the later mankind will gain the conception that it must have." (Lecture of 1 October 1916 [1])

Rudolf Steiner's statement that the earth crosses the paths of the other planets becomes more understandable if one takes into account the extent of Mercury's lemniscate depicted above. Close to the passage of Mercury through the Perihelion and Aphelion, Mercury's lemniscate occupies a position in which it extends beyond the Copernican Earth orbit (cf. Figures 150, 152 to 154 and 156). The inclination of Mercury's lemniscate towards the ecliptic is the same as that of the Copernican Mercury orbit, for the latter is a lemniscate half. The slope is $7^{\circ}$, while the Earth orbit is considered to be at an inclination of $0^{\circ}$ to the ecliptic in the Copernican view. This circumstance would prevent Mercury's path from being crossed by the Earth. But as the paths advance upward (also Copernican in the sense of the apex movement of the sun), there is the possibility for the earth, at certain times to cross over Mercury's path running over it.

### 3.2.2 The Lemniscate of Venus

For Venus too, the first step in positioning its lemniscatory path is to first correctly align its Copernican orbit in the zodiac. In contrast to the strong elliptical Mercury orbit with an eccentricity of 0.2056 , the Venus orbit is nearly circular.


Figure 159: Orientation of the Venus orbit in the zodiac

It has the lowest eccentricity with 0.0068 and is therefore the most harmoniously shaped of all planetary orbits of our solar system. If one draws a straight line from the furthest point from the sun (aphelion) of Venus' orbit to the nearest point of the sun (perihelion), it points in the year 2011, seen from the sun, to about $12^{\circ}$ Leo (Figure 159).

Here, too, the question arises as to whether the Copernican orbit of Venus describes its original impulse of motion or whether it is the result of a rotating lemniscatory path. Does the lemniscate of Venus rotate at exactly $60^{\circ}$ per month as it is so similar to the almost circular Earth orbit, and does it also rotate around its own axis for half a year? Does it also rest for another half year afterwards?

Figure 160 shows the presumed lemniscate of Venus. This is also based on the assumption that the orbit point of the perihelion corresponds to the midpoint of the lemniscatory path. The upper (thinner) lemniscate half of Venus' path corresponds to the Copernican Venus orbit at the depicted time. Venus passed through its perihelion or its lemniscate midpoint on 21 February 2009. Viewed from the earth, it was in $12^{\circ}$ Aries at that time. Venus needs 224.7 days for a full round on its Copernican orbit. This period corresponds to its "ellipse year" or, more appropriately in this case, "Copernican year", since the elliptical deformation is so low. For a full round on the marked lemniscatory orbit, Venus needs twice the time, about 449 days. A lemniscatory year thus corresponds to two Copernican years. A lemniscatory month of Venus is about 37.5 days.

Figures 160 to 166 show the trajectory of Venus when its lemniscate is allowed to rotate $60^{\circ}$ per month like the Earth-Sun lemniscate, again for only half a year. Venus' lemniscate, however, must rotate in the opposite direction like Mercury's lemniscate. Its rotation angle is therefore not $+60^{\circ}$, but $-60^{\circ}$. In total, in half a lemniscatory year of about 225 days, a full $360^{\circ}$ rotation of Venus' lemniscate takes place. Venus runs clockwise on its lemniscatory path, but at the same time seems to run counter-clockwise on its Copernican orbit. The latter is just the outer appearance that results from the interaction of planetary lemniscatory motion and orbital rotation.


Figure 160: Position of Venus' lemniscate on 21 February 2008


Figure 161: Position of Venus' lemniscate on 31 March 2009


Figure 162: Position of Venus' lemniscate on 7 May 2009


Figure 163: Position of Venus' lemniscate on 13 June 2009


Figure 164: Position of Venus' lemniscate on 21 July 2009


Figure 165: Position of Venus' lemniscate on 28 August 2009


Figure 166: Position of Venus' lemniscate on 5 October 2009

On 5 October 2009 Venus regains its perihelion in the lemniscate midpoint (Figure 166). From now on, Venus' lemniscate rests for half a lemniscatory year, and Venus continues to run counterclockwise, in accordance with the change of direction in the lemniscate. During this period the lemniscatory path and the Copernican path completely coincide. Only after 225 days, when Venus reaches again its perihelion in the lemniscate midpoint, the lemniscate of Venus will rotate again for half a year. The same regularity has been found already in the Sun-Earth lemniscate as shown above (see table on page 114).

The lemniscate of Venus is also tied to the common Earth-Sun lemniscate via its lemniscate midpoint. Most of the time, however, this does not lie on it, as the Earth-Sun lemniscate constantly changes its position. Even when it "rests", it is subject to the year-round continuous rotation. Only occasionally in the course of a solar year a contact can occur between the Earth-Sun lemniscate and the Perihelion point or lemniscate midpoint of an inner planet. Rudolf Steiner drew a sketch of the time of the equinoxes, when the sun is at the midpoint of its path and the earth is at its farthest point (cf. Figure 145, page 111). Then, the Earth-Sun lemniscate not only comes close to the lemniscate midpoint of the path of Mercury, but also to that of the path of Venus, as shown in Figure 167.


Figure 167: Positions of the lemniscate midpoints of the paths of Mercury and Venus at the vernal equinox

The lemniscatory paths of Mercury and Venus are not marked there, because they do not follow the rhythm of the solar year and take different positions at the equinoxes of the successive years. The lemniscate midpoints of the paths of Mercury and Venus are located in the immediate vicinity of the Earth-Sun lemniscate or on it, if a certain width is granted. It does not need to be a contradiction that the two lemniscate midpoints (red and green) do not lie on the same side of the Earth-Sun lemniscate. Rudolf Steiner possibly only wanted to show in principle the situation at the equinoxes and the connection of the inner planets to the Earth-Sun path. He also marked both paths much too small. On such small lemniscatory paths, Mercury and Venus could not occupy those
positions around the Sun as they are provably by Copernicanism and lead to the impression of a simple orbit of the planets around the Sun. In addition, in Rudolf Steiner's sketch, the lemniscate midpoint of Mercury was also moved too close to the sun, as shown in Figure 145. In order to maintain the distance between Mercury and the Sun it can only lie on the Copernican Mercury orbit. Thus, it is obviously a purely conceptual sketch of Rudolf Steiner, which nevertheless expresses the essentials.

Finally, let's take a look at the proportions of the lemniscatory paths of the inner solar system. In Figure 168 shows the lemniscate of Mercury in the position of 28 January 2008, just before it reached its vertical position. Mercury was passing through its perihelion. At this time, the upper half of the lemniscate or Mercury is always congruent with the Copernican elliptical orbit of Mercury (cf. Figure 156, page 119). The position of the lemniscate of Venus corresponds to 31 March 2009 (cf. Figure 161, page 124). It shows that in its rotations the lemniscate of Venus protrudes far beyond the Copernican Mars orbit (dashed red). It can lead away the endpoints of its two lemniscate halves (aphelion points) up to 2.164 AU from the sun, calculated from the perihelial distance of Venus-Sun ( 0.718 AU ) + longitudinal diameter of a lemniscate half (= perihelial distance $0.718+$ aphelial distance $)=0.728$ AU. The Earth-Sun lemniscate is depicted at the time of the autumnal equinox. It lies horizontally in the illustration (blue and orange). It can be clearly seen that the lemniscate of Mercury is the path closest to the sun and the earth. If one understands Mercury not only as the physical planet, but also the entire efficacy of power along its path, Rudolf Steiner's statement becomes understandable: "... Mercury is closer to the earth than the other planets". (Lecture of 14 April 1912 [14])


Figure 168: Proportions of the orbits and paths of the inner solar system

### 3.3 The movements of the Earth-Sun Lemniscate, seen from the north ecliptic pole

In Rudolf Steiner's sketch of the Earth-Sun lemniscate (Figure 147, page 113), the sun is shown running in a clockwise direction. Therefore, in the considerations based on it, the zodiac also had to be presented in a clockwise direction. About the reason why Rudolf Steiner chose this direction of movement of the sun, only assumptions can be made. It is conceivable that he observed the movement from the south ecliptic pole. But from this follows the question of why he should have chosen this completely unusual point of view, which in the end only makes the process of cognition more difficult for the viewer of his sketches. Another reason therefore seems more likely. Rudolf Steiner had the capability to perceive imaginatively in the astral world and to transfer the images gained there into earthly awareness. Imaginations are experienced mirrored in comparison to the sensory perceptions of the physical world. Even imaginatively viewed dates must be reversed before being applied to the physical world. Rudolf Steiner obviously left this task of reversing the sketches mirror-inverted to us. However, it could only be done after the motion sequences readable from the original sketches had been understood to a certain extent. The positions of the sun and earth on their lemniscatory paths at the solstices and equinoxes resulting from the reversal can be seen in Figure 169. From the winter solstice, the sun and the earth first run counter-clockwise.


Figure 169: Solstices and equinoxes in the lemniscate

Figures 170 to 182 show the motion sequences already depicted in PART 2 for the formation of the cosmic crosspath of the sun and the earth, but now also in mirror-image inversion, with the zodiacal order in counterclockwise direction. In addition, it was taken into account that with the discovery of the separate half-yearly rotations of the lemniscatory paths of Sun and Earth the term "pivoting motion" is no longer necessary. There are only rotations. In the figures, therefore, the terms originally used "pivoting motion" and "rotary motion" have been replaced by the unitary term "rotation" (according to the table on page 114).


Figure 170: Position of the Earth-Sun lemniscate on 22 December - Sun in $0^{\circ}$ Capricorn


Figure 171: Position of the Earth-Sun lemniscate on 21 January - Sun in $0^{\circ}$ Aquarius


Figure 172: Position of the Earth-Sun lemniscate on 19 February - Sun in $0^{\circ}$ Pisces


Figure 173: Position of the Earth-Sun lemniscate on 20 March - Sun in $0^{\circ}$ Aries


Figure 174: Positions of the Earth's and the Sun's lemniscate in relation to each other on 20 April - Sun in $0^{\circ}$ Taurus


Figure 175: Positions of the Earth's and the Sun's lemniscate in relation to each other on 21 May - Sun in $0^{\circ}$ Gemini


Figure 176: Position of the Earth-Sun lemniscate on 21 June - Sun in $0^{\circ}$ Cancer


Figure 177: Position of the Earth-Sun lemniscate on 22 July - Sun in $0^{\circ}$ Leo


Figure 178: Position of the Earth-Sun lemniscate on 23 August - Sun in $0^{\circ}$ Virgo


Figure 179: Position of the Earth-Sun lemniscate on 23 September - Sun in $0^{\circ}$ Libra


Figure 180: Positions of the Earth's and the Sun's lemniscate in relation to each other on 23 October - Sun in $0^{\circ}$ Scorpio


Figure 181: Positions of the Earth's and the Sun's lemniscate in relation to each other on 22 November - Sun in $0^{\circ}$ Sagittarius


Figure 182: Position of the Earth-Sun lemniscate on 22 December - Sun in $0^{\circ}$ Capricorn

Even after reversing the zodiac order, the sun's positions on the cosmic crosspath remain unchanged. Only the earth's positions change from left to right or from right to left:


Figure 183: The annual positions of Sun and Earth in the cosmic crosspath

From a three-dimensional perspective, the cosmic crosspath of the sun and earth runs as shown in Figure 184. It is inclined sideways to the left by $23.5^{\circ}$. Considering the additional vertical advancing of the lemniscatory paths, including the regulating motion of the lemniscate axis, the sun's six-month forward and backward movement on the "cross stem" changes into an ascending zig-zag path as shown in Figure 185. The earth's path also runs in zigzag. It cuts the sun's path at the two solstices.


Figure 184:
The inclined non-advancing cosmic crosspath of Sun and Earth


Figure 185:
The inclined vertical advancing cosmic crosspath of Sun and Earth, taking into account the lemniscatory axis motion

### 3.4 The moving cosmic crosspath and the resting sun

Although the zigzag paths of the sun and the earth were consistently determined from Rudolf Steiner's sketches of paths and movements, it is difficult to imagine that a planet like the earth or even the sun, which is many times larger, should run on such angular paths. In addition, we have a statement by Rudolf Steiner, according to which the inner planetary system follows the outer planets or they drag it behind them. The Sun must therefore be bound to Mars in some way. But how is that supposed to happen when it runs forward and backward on its zigzag path in sixmonthly cycles? A connection to Mars really requires a balancing or counteracting force as it is also required for the vertical advancing of lemniscatory paths. The lemniscatory axis movement completely compensates the lifting and tilting processes associated with the vertical advancing that they no longer appear outwardly. The result is an equilibrium or resting state of the ecliptic and the north celestial pole, which is caused by several forces. Similarly, for the half-year forward and backward movement of the sun in the cosmic crosspath or zig-zag path, a force acting exactly opposite must be assumed so that the sun, as a result, appears to be resting in the middle. This forward-backward or zig-zagging force would pull the cosmic cross path of the sun and the earth just as much forward as the sun travels backwards on the cosmic cross stem and pull it back just as much as the sun runs forward. The idea of such a counter-regulating force may seem at first even
more absurd than the zigzag paths of the sun and the earth. Ultimately, however, such a force is not further removed from today's conventional physical concept of planetary motion than the counter-regulating lemniscatory axis movement described by Rudolf Steiner. Already the existence and the rotation of the lemniscatory paths will undoubtedly far exceed the imagination and acceptance of most of today's astronomers. Indeed, even with a certain right, because today's physics knows only gravity, centrifugal force and rotation as the only path-forming forces of the planets, with which such paths cannot be explained. The reverse rotation of the Earth's axis, as described by Copernicus as a "third earth movement", is also rejected by modern astronomy (cf. pages 86 and seq.)

The exact sequence of events, how the sun comes to rest by the forward and backward movement of the cosmic crosspath and how as a result the earth takes up positions in the course of the year which, when joined together by a line, form an ellipse, is shown in Figures 186 to 192. Only as the final result of a multistage path-forming process does the Copernican Earth orbit (blue) develop, as can be observed physically. However, the elliptical shape shown in the figures does not reflect the different perihelion-aphelion distances of the earth. The resulting slightly elliptical deformation of the circular path is far too small to represent it with the scale ratios of such a figure. The fact that the Copernican orbit appears elliptical or ellipse-like in the figures is due to the inclination of the earth's orbit by $23.5^{\circ}$. The view is again from the north ecliptic pole to the ecliptic plane. The left half of the Copernican earth orbit is to be considered to run below the plane of the paper, the right half as running above it (cf. Figure 184). Due to this inclination, the Copernican orbit in the figures seemingly becomes an ellipse.

20 Mar and 23 Sep
Vernal and Autumnal equinoxes


Figure 186: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit for the vernal and autumnal equinox with the moving cosmic crosspath and "resting" Sun

The forward-backward force pushes or pulls the cosmic crosspath of the sun and the earth always exactly as far back and forth (or in the figures up and down) that the sun remains in a state of equilibrium in the middle of the earth's orbit and seems to be at rest. In the process, a part of the yellow cross stem of the cosmic cross path gradually moves beyond the upper edge of the picture or later below the lower edge of the picture. The monthly positions of the earth (blue) on the Copernican orbit and of the sun (orange) on the cross stem are each transferred to the following
figures in gray. A comparison of figures 186 and 187 shows how the sun initially runs down on the yellow cross stem (orange arrow), but ultimately "rests" in the middle of the Copernican orbit, because at the same time the cosmic cross path moves up this very distance. Although the sun reaches its next month's position, it is still in the middle of the Copernican orbit. Due to the symmetry of the earth's orbit, two monthly positions can always be displayed in one figure.


Figure 187: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit on 20 April and 23 August with the moving cosmic crosspath and "resting" Sun


Figure 188: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit on 21 May and 22 July with the moving cosmic crosspath and "resting" Sun


Figure 189: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit for the summer solstice on 21 June with the moving cosmic crosspath and "resting" Sun


Figure 190: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit on 19 February and 23 October with the moving cosmic crosspath and "resting" Sun


Figure 191: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit on 21 January and 22 November with the moving cosmic crosspath and "resting" Sun


Figure 192: The positions of Sun and Earth on the cosmic crosspath and on the Copernican orbit for the winter solstice on 22 December with the moving cosmic crosspath and "resting" Sun

All considerations taken together give the following result: Rudolf Steiner's sketches and suggestions for a more realistic consideration of the planetary motion did not aim to deny the Copernican observations, but to guide astronomy from an understanding of form to an understanding of movement. Similar to the way in which living beings on earth secrete a layer of dead cells as the end product of their ethereal formative processes and form a physically visible, external form with them, also in the planetary motion the various path-forming forces ultimately lead to the outer form of a physically observable planetary orbit. This insight is reminiscent of a statement by Rudolf Steiner from the first lecture of the Berlin cycle entitled "Human thought and cosmic thought" ("Der menschliche und der kosmische Gedanke", Lecture of 20 January 1914 [15]). At that time, he used the example of the term "triangle" to explain how one can ascend from a "special thought" to a "general thought", moving from thinking in forms to thinking in movements.
"If we want to rise from the specific thought to the general thought, we are challenged to bring the specific thought into movement in such a way that the thought in movement is the general thought, which slips from one form into another. I say 'form'; the correct thought is: the whole is in movement, and every single thing that comes out by the movement is a self-contained form. ... Now I am drawing something - actually I am not drawing it, as I already said, but you can imagine it -, which is to evoke the idea, that the general thought is in movement, and brings forth the single form by its standing still - 'creates the form', I say.

Hence we see: the philosophers of Nominalism, who necessarily stand at a boundary-line, move within a certain realm, the realm of the Spirits of Form. Within this realm of the Spirits of Form, which is all around us, forms are prevalent; and since the forms are prevalent, in this realm we find single, strictly self-contained separate things. From this you can see that the philosophers, whom I mean, have never made the decision to leave the realm of forms and can therefore have nothing else in the general thoughts but words, really mere words. If they would leave the realm of special things, i.e. forms, they would enter into an imagination that is in continuous movement, i.e. they would have in their thinking a realization of the realm of the Spirits of Movement, the next higher hierarchy. And when in recent times of Western thinking there was one who deigned to really think in this way, he was little understood, although much was said and much babbled about him. Look it up, what Goethe wrote in his "Metamorphosis of Plants", what he called the 'primal plant' (Urpflanze) ..."

In his instructions on the further development of art, Rudolf Steiner also emphasized that the form be born out of colour. Likewise, eurythmy was a means for him to illustrate the varied living, moving processes, from which ultimately the visible forms result. Obviously, the shape of the Copernican orbit with the sun in the middle is only the final result of a whole series of preceding movements due to the interaction of diverse formative forces of the planetary paths. Even if the Earth's orbit ultimately appears circular or elliptical, the earth still does not run around the sun according to Rudolf Steiner's statements and sketches, but follows a lemniscatory impulse of movement.

How it is possible that the positions of the earth ultimately come to be on a circular path (elliptical orbit) inclined by $23.5^{\circ}$, shall be illustrated in Figures 193 to 205 in monthly steps finally and summarizing. It can be seen there clearly:

1. The lemniscatory movements of the sun and the earth
2. The positions or rotations of their lemniscatory paths
3. The straight-linear pushing back and forth of the cosmic crosspath or the midpoint of the Earth-Sun lemniscate. This midpoint travels up the purple midline for half a year and then down again for half a year.


Figure 193:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath at the winter solstice with "resting Sun"


Year-round continuous rotation of the double-lemniscate system by - $30^{\circ}$

Figure 194:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 21 January with "resting Sun"


Figure 195:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 19 February with "resting Sun"


Figure 196:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath at the vernal equinox with "resting Sun"


Figure 197:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 20 April with "resting Sun"


Figure 198:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 21 May with "resting Sun"


Figure 199:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath at the summer solstice with "resting Sun"


Figure 200:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 22 July with "resting Sun"


Figure 201:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 23 August with "resting Sun"


Figure 202:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath at the autumnal equinox with "resting Sun"


Figure 203:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 23 October with "resting Sun"


Figure 204:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath on 22 November with "resting Sun"


Figure 205:
The position of the earth on the lemniscatory path and the Copernican orbit, the orientation of the earth's and the sun's lemniscate and the position of the moving cosmic crosspath at the winter solstice with "resting Sun"

Now that all the motions in the lemniscatory path system can be reconciled with the Copernican system, one can try to reproduce them in three dimensions as they would be imagined in space. Figure 206 shows the sun in the center of the Copernican Earth orbit at the time of the winter solstice on 21.12. of a year. The earth's axis points vertically upwards to the north celestial pole. The purple line connecting the earth's position at the winter solstice with the opposite position at the summer solstice corresponds to the purple line in the figures 193 to 205. Along this line, the lemniscate midpoint of the double-lemniscate path of Sun and Earth (which is also the midpoint of the cosmic crosspath) moves to the left down for half a year, and then to the right up for half a year.

In the three winter months, both lemniscates initially rotate together in the clockwise direction, whereby the individual earth positions come to lie on the light blue elliptical line (Figures 206 to 209). This creates the impression that the earth would simply run around the sun on its so-called Copernican orbit. Instead, the earth moves on its rotating lemniscatory path. The sun, on the other hand, seems to be standing still in the center of the Copernican orbit (or moving only along the red arrow line towards the solar apex), even though it also moves along its own lemniscatory path.

From the vernal equinox on 20 March until the summer solstice on 21 June only the Earthlemniscate turns clockwise by $+30^{\circ}$ per month. The sun's lemniscate rests. However, due to the continuous, year-round rotation of the double-lemniscate system by - $30^{\circ}$ per month counterclockwise, it is turned back to a position which it had already occupied at the winter solstice (Figures 209 to 212).


Figure 206: Double-lemniscate path and Copernican orbit on 21 December (winter solstice)


Figure 207: Double-lemniscate path and Copernican orbit on 21 January. Earth's position retained from 21 December.


Figure 208: Double-lemniscate path and Copernican orbit on 19 February. Earth's positions retained from 21 December to 21 January.


Figure 209: Double-lemniscate path and Copernican orbit on 20 March (vernal equinox). Earth's positions retained from 21 December to 19 Februray.


Figure 210: Double-lemniscate path and Copernican orbit on 20 April Earth's positions retained from 21 December to 20 March.


Figure 211: Double-lemniscate path and Copernican orbit on 21 May. Earth's positions retained from 21 December to 20 April.


Figure 212: Double-lemniscate path and Copernican orbit on 21 June (summer solstice). Earth's positions retained from 21 December to 21 May.

Until the summer solstice on 21 June, the two lemniscates have turned around $180^{\circ}$ to each other. While the sun's lemniscate has turned back in the spring months, the earth's lemniscate has continued to rotate clockwise continually $+30^{\circ}$ per month over the course of half a year, i.e. by $6 \times 30^{\circ}=180^{\circ}$.

In the following quarter, both Lemniscates rest. They are only turned back - $30^{\circ}$ per month counterclockwise by the year-round continuous rotation of the double-lemniscate system. The midpoint of the Earth-Sun lemniscate moves towards the sun in the center of the Copernican Earth orbit. On 23 September, at the autumnal equinox, both lemniscates occupy a position similar to that of the vernal equinox. However, due to the turns, on the side facing the observer, the earth's lemniscate now lies above the sun's lemniscate (Figures 212-215).


Figure 213: Double-lemniscate path and Copernican orbit on 22 July. Earth's positions retained from 21 December to 21 June.


Figure 214: Double-lemniscate path and Copernican orbit on 23 August.
Earth's positions retained from 21 December to 22 July.


Figure 215: Double-lemniscate path and Copernican orbit on 23 September (autumnal equinox). Earth's positions retained from 21 December to 23 August.

From the autumnal equinox, the sun's lemniscate rotates clockwise by $+30^{\circ}$. The earth's lemniscate rests for a quarter of a year. However, it is turned back even further by the year-round continuous rotation of the double-lemniscate system by $-30^{\circ}$ per month counter-clockwise, so that at the winter solstice both lemniscates take up their positions again as they did a year ago (Figures 215 to 218). The lemniscate midpoint, which is also the midpoint of the cosmic crosspath, moves along the purple line towards the earth's winter solstice position on 21 December.

Figure 218 shows that all twelve monthly Earth positions are located on the Copernican Earth orbit. Since the Earth's axis is vertical in the figures, the Copernican Earth orbit is inclined by $23.5^{\circ}$. This inclination ultimately causes the inclination of the Sun's path in the sky or the "ecliptic plane" from a geocentric point of view.


Figure 216: Double-lemniscate path and Copernican orbit on 23 October. Earth's positions retained from 21 December to 23 September.


Figure 217: Double-lemniscate path and Copernican orbit on 22 November.
Earth's positions retained from 21 December to 23 October.


Figure 218: Double-lemniscate path and Copernican orbit on 21 December (winter solstice).
Earth's positions retained from 21 December to 22 November.

### 3.5 The five formative steps of the Earth's path

In the lecture of 29 April 1908 [10] Rudolf Steiner described the path of the earth in the cosmos with the following words: "In reality, the sun races at great speed through space towards the constellation of Hercules. Such a movement, as it is usually described, is only faked by the fact that the planets also move along with it. The true path of the Earth forms a helical line. What is called the obliquity of the ecliptic is the line of gravity between Sun and Earth ..." With this statement "The true path of the earth is a helical line" Rudolf Steiner is in complete harmony with the prevailing astronomical opinion about the path of the earth in the cosmos. In Copernican terms, the helix is created by the earth following the sun, with its elliptical orbit, while the sun races in the direction of the solar apex.

On the other hand, despite the agreement on the outer shape of the Earth's path, Rudolf Steiner deviates completely from the prevailing astronomical opinion when it comes to explaining the creation of this form. He repeatedly contradicts the view that the elliptical orbit or helical path would come about by a rotation of the earth around the sun. Also regarding the paths of the other planets he has a different opinion. In the conferences with the teachers of the Freie Waldorfschule in Stuttgart he explained this topic: "The helical line continues in space. So, it is not that the planets move around the sun, but these three: Mercury, Venus, Earth, follow the sun, and these three: Mars, Jupiter, Saturn go ahead." (Conference of 25 September 1919 [4]). Three years later he explained what the paths of the planets really should look like: "We have a system of lemniscates arranged in certain way as the paths of the planets and also as the Earth-Sun path." (Lecture of 17 January 1921 [3]) Figure 219 shows for the earth, how its helical path in the cosmos is shaped by the interaction of the vertical and horizontal advancing of its lemniscatory path, which is bound into the double-lemniscate system Earth-Sun.


Figure 219: The formative process of the helical path of the Earth by the vertical and horizontal advancing of the double-lemniscate system of Sun and Earth

Obviously, Rudolf Steiner was not interested in negating the physically verifiable Earth's orbit. Rather, he focused his attention on a whole series of different path-forming processes and pathforming forces, only through the interaction of which the Copernican Earth's orbit arises. A synopsis of all his statements and sketches, as well as of the resulting consequences, leads to the conclusion that the Earth's orbit has to go through different formative steps before the outer appearance of its helical path is created as the fifth and final step.

Figure 220 summarizes this process of creating the Earth's helical path over five formative steps. The starting point is the lemniscatory planetary movement according to Rudolf Steiner's statement already quoted above: "We have a system of lemniscates arranged in certain way as the paths of the planets and also as the Earth-Sun path." This original trajectory is subjected to circular forces in a second formative step, which lead to the fact that the sun, seen from the earth, runs on a circular path in the sky. As a further consequence the third formative step of the cosmic crosspath of Sun and Earth is created. It represents a peculiarity of the double-lemniscate system Earth-Sun. By further forces acting on this path in a fourth formative step, which push the path back and forth on a straight line, the sun ultimately comes to rest and the connecting line between the positions of the earth over the year emerges as the Copernican orbit. This is winded up to a helix in a fifth formative step, as the sun with all planets belonging to it move in the direction of solar apex.


Figure 220: The five formative steps of the Earth's path

Interestingly, the course of the formative process indicated by four arrows in Figure 220 again yields a section of a lemniscate. The sequence of movements takes place first in a clockwise direction (from the lemniscate over the circle to the cross) and then counterclockwise (from the cross over the straight line to the helix). The arrangement of the five formative steps is by no means arbitrary. Of course you could also simply let them follow one after the other on a straight line. However, there are relationships between these geometric basic forms, which at first sight may not be immediately apparent. The four formative steps Lemniscate, Circle, Straight Line and Helix were place opposite each other in the manner shown, because they form pairs of opposites. The circle forms a contrast to straight line or to its radius. The lemniscate and the helix also form a pair of opposites. Both result from the interaction of circular force and straight force, but in a contrary way. Submitting a circle to a continuously acting straight vertical force produces a helical shape (see Figure 219). Since the effectiveness of the circular force is maintained, the circular path is not resolved, but pulled apart upwards only. If, on the other hand, you subject a circle to a straight horizontal force, the circular path is broken open in one direction. But as soon as in the
course of the breaking process there is an overstretching at the straight line, the temporarily inferior circular force strives for the next possible point for a return to the circular form, only now in the other direction. This creates a lemniscate's form. A further contradiction between lemniscate and helix is that in the case of the helix the vertical force acts continuously from the beginning, whereas in the lemniscate only the circular force is active at first and only after a first circle as been formed the straight vertical force intervenes and continues to act until a second circular formation in the reverse direction is released. Thus, a lemniscate has a "passive" half, in which it follows undisturbed its underlying circular force, and an "active" half, in which it constantly has to struggle against the horizontal force and to maintain its circular shape. For this reason, helix and lemniscate are juxtaposed in Figure 220 as a pair of opposites. Another special feature is that the cross representing the cosmic crosspath is the center of the lemniscatory process of formative steps (light gray lemniscate), just as in the preceding figures the midpoint of the cosmic crosspath coincides with the lemniscate midpoint of the Earth-Sun path.

In general, Figure 220, with its four basic principles and a fifth in the middle, recalls another image, which we all know from the description of the creatures around God's throne in the Revelation of St. John: "and in the middle of the throne and around the throne are four living creatures ... The first creature is like a lion, the second is like an ox, the third being has a human face and the fourth is like a flying eagle. ... And I saw in the midst of the throne and the four creatures ... a lamb standing slain, as though it had been slain." (Revelation 4: 6-7 and 5: 6) Thus we find in and around God's throne the spiritual archetypes of the five formative steps of the Earth's path, where the lamb takes the place of the cross or the cosmic crosspath of Sun and Eearth, those two planets, which are the most important domains of the Lamb.


Figure 221: The four living creatures around the throne of God
Copernican astronomy, with the elliptical orbit and the helical path of the Earth, or of the other planets also, captures only the ultimately emerging outer forms from the whole complicated formative process of the planetary paths, which is apparently based on the five archetypes or primal forces in the spiritual Sun. The further development of astronomy now consists in advancing to the formative steps and formative forces that underlie the outer forms. Thus Rudolf Steiner's words from the lecture of 1 October 1916 become understandable: "Now you have the external image, the purely geometrical external image; the other image will be added, and it is only from the union of the two images that later humanity will gain the idea that it must have." [1]

### 3.6 The twelve formative forces of the Earth-Sun path

In the course of the previous considerations, a whole series of very diverse forces has been described that are involved in the formative process of the planetary orbits. These forces shall be juxtaposed with each other in the following and their respective share in the interaction of all formative forces shall be explained. It has to be taken into account that the paths of the sun and the earth, which together form a double-lemniscate system, are a peculiarity and require more formative forces than the paths of the other planets for their final form. All in all, apparently twelve different forces are involved in the path formation. They can be grouped into four groups of three forces each:

## The three lemniscatory forces:

- The basic planetary movement in a closed lemniscatory path.
- The vertical advancing of the lemniscatory path.
- The compensating or counter-regulating lemniscatory axis movement, which is necessary due to the vertical advancing, in order to maintain the course of the Sun at $0^{\circ}$ ecliptic latitude and the orientation of the earth's axis on the north celestial pole.


## The three circular or rotational forces:

- The half-yearly separated rotations of the sun's lemniscate and the earth's lemniscate.
- The year-round continuous rotation of the double-lemniscate system.
- The compensating or counter-regulating circular reverse rotation of the earth's axis, which is necessary due to the year-round continuous rotation, in order to maintain the orientation of the earth's axis towards the north celestial pole ("Third Copernican Law", see page 85 ff .). The orientation of the sun's axis must also be "corrected" accordingly if it is to remain unchanged in its relation to the earth's axis.


## The three straight-linear forces:

- The straight-linear pushing forward and backward of the cosmic crosspath of Sun and Earth, to bring the sun to rest (canceling the zigzag path).
- The straight-linear horizontal pulling forward of the rotating lemniscatory paths (horizontal advancing).
- The compensating or counter-regulating straight-linear backward inclination of the earth's axis, which is necessary due to the horizontal advancing, in order to maintain the orientation of the earth's axis towards the north celestial pole. The orientation of the sun's axis must also be done in a similar manner if it is to remain unchanged in its relation to the earth's axis.

In the resulting appearance of the planetary motion in the cosmos, three more forces can be observed, which are of a superordinate kind.

## The three superordinate basic forces:

- Stability and fixation of the earth's axis.
- Variability and incomprehensibility.
- Balance and target orientation (Apex).

From this grouping of four times three forces results the question as to whether there is a connection between these twelve formative forces of the planetary orbits and the twelve signs of the zodiac. Of course, one has to be very careful not to speculate on any connections and to improperly cram the forces incorrectly into the system of the zodiac. Nonetheless, the attempt should be dared in the following to establish such relationships. The starting point is "the first living creature" before God's throne, which is "like a lion". A comparison of Figure 221 with Figure 220 shows that this creature corresponds to the basic principle of a lemniscate. On the other hand, as it also represents the summer season in the course of the year and thus represents the three summer signs of Cancer, Leo and Virgo, it is reasonable to assume that the three summer signs are associated with the three lemniscatory forces mentioned above. In the following considerations on the twelve cosmic formative forces Rudolf Steiner's statements on the zodiacal forces, the twelve primal consonants corresponding to them as expressions of the creative Cosmic Word and their illustration in eurythmic gestures are taken into account.

## THE THREE LEMNISCATORY FORCES:

## - The self-contained lemniscate - VIRGO:

According to Rudolf Steiner, the lemniscate is the starting point and the primal form of all planetary motion. As a firmly self-contained form, it is in a relationship to Earth sign Virgo which serves the enveloping shaping. According to Rudolf Steiner, this force of enclosure also manifests itself in the consonant B . In its closed form, this shows a clear similarity to the number 8 or to a vertical lemniscate, so that a relationship between the two can be easily established.

## - The vertical advancing of the lemniscate-LEO:

However, the self-contained lemniscate is subject to a metamorphosis. By a second force it is opened and continued in the vertical direction. This causes a continuous lifting of the lemniscate. Rudolf Steiner made the corresponding force visible to us in the eurythmic gesture of the sound " T ". The vertically standing human being as expression of the vertical lifts both arms (the two lemniscate halves) and closes them together in the middle above his head, so that a horizontal 8 is formed. This gives the picture of a lemniscate lifted to the top. The crossbar of the letter T can be thought to emerge from a flattened horizontal lemniscate.

## - The lemniscatory axis movement - CANCER:

The vertical advancing of the Lemniscate requires a lemniscatory axis movement as a balancing and counteracting force. It allows, despite the vertical advancing, the Sun to run in $0^{\circ}$ ecliptic latitude and the Earth's axis to remain aligned on the north celestial pole. The consonant F, which Rudolf Steiner assigns to the zodiac sign Cancer, nowadays does not show a formal connection with a lemniscate. The Greeks use the letter $\Phi$ (Phi) for this sound. This style of writing still reveals clearly the connection with the lemniscate. The vertical bar may be interpreted as a vertical axis, which is subjected to a lemniscatory movement, as indicated by the horizontal lemniscate. The latter is in the middle position on the vertical axis, as if to indicate that a state of equilibrium is being created here.


Figure 222: The twelve formative forces of the Earth-Sun path

## THE THREE CIRCULAR OR ROTATIONAL FORCES:

## - The half-yearly separate rotations - GEMINI:

After the three lemniscatory forces have been effective, the planetary paths are subjected to rotations. In order for the sun and the earth to be able to maintain the distance between themselves on their course, their lemniscatory paths have to perform half-yearly rotations, partly individually and partly together. Both lemniscates always make a full $360^{\circ}$ rotation in half a year, i.e. $+60^{\circ}$ per month in a clockwise direction. As a result, seen from the earth, the sun seems to stand still in the zodiac. The phenomenon of the two separate and yet coordinated rotations could also be called a "twins rotation". In the letter H with its two vertical lines, which are connected to each other in the middle, it is symbolized that the two forces work together. Instead of an H at the beginning of a word, the Greeks write the spiritus asper, which resembles a small semicircle. It can be understood as a symbol of the fact that the half-yearly rotation of the earth's lemniscates is only one half of a composite rotational process, which still requires the half-yearly rotation of the sun's lemniscate to become completed.

## - The year-round continuous rotation - TAURUS:

In order for the sun to move through the zodiac as seen from the earth, another force must act on the lemniscates of Sun and Earth, which rotate at half-yearly intervals. In addition, it requires a year-round continuous rotational force, which turns the entire double-lemniscate system of Sun and Earth by $-30^{\circ}$ per month counterclockwise, thereby guiding the sun through the zodiac. We find it symbolized in the consonant R , the sound of the force of the zodiac sign Taurus. It is no coincidence that the word "round" begins with an R, just as the words "rotation" * or "roll". They describe onomatopoeically the interaction of a circular force. The Latin $R$ has clearly retained the circle as the underlying principle of form in its upper half, even the Greek Rho ( P or $\rho$ ), although it is more like the Latin P in its style of writing.

## - The circular reverse rotation of the earth's axis - ARIES:

Again, the third force must be a regulating force that stabilizes the earth's axis. It must counteract the continuous annual rotation of the earth's axis. Apparently the axial forces always belong to a cardinal sign. Already the lemniscatory axis movement had to be assigned to the cardinal sign Cancer. Here it is now the cardinal sign Aries, which regulates the axis position. According to Rudolf Steiner, the formative power of Aries manifests itself in the consonant W , which is written by hand not as angular as in print type, but is composed of two incomplete, opposing circular movements.** In words such as vault, winding and waves or swell, the "W" of "V" describes the affiliation to the circular forces. In the winding horn of Aries, we have a beautiful image for this force of circular backward rotation.

The circular reverse rotation of the earth's axis was already described by Copernicus almost 500 years ago as the "Third Movement of the Earth". As the first movement, he called the daily movement of the earth (rotation), as the second the supposed annual course of the earth around the sun (circular orbit). The third movement of the annual circular reverse rotation of the earth's axis, described by him, has been eliminated from astronomy in the course of its historical evolution, so that it has been completely forgotten. Copernicus emphasized that this movement was necessary for the earth's axis to take up its positions in relation to the sun, which are essential for the creation of the seasons. He had also recognized that the circular reverse rotation of the earth's axis is slightly less than a full rotation and that the phenomenon of precession arises as a result, i. e. the slow reverse movement of the vernal point through the zodiac (see section 2.4, page 86 et seq.). The vernal point belongs to the first vernal sign, which shows the affiliation of this precession-causing force to the zodiac sign Aries. Rudolf Steiner called the discovery of Copernicus the "third Copernican law" and pointed out that it will be of importance again in the future.

The interplay of the first six formative forces creates the cosmic crosspath of Sun and Earth. It is certainly no coincidence that it comes about just under the spring and Easter sign of Aries. The cross of death and the resurrection of the Lamb are at the centre of all the formative forces of the planetary paths. It could be argued here that the cosmic crosspath arises already after the influence of the force of the zodiacal sign Taurus, the continuous year-round rotation. But it immediately requires the simultaneous intervention of the counter-regulating force Aries on the earth's axis. In general, we should think of all forces rather as acting simultaneously, although on the other hand the individual forms of the paths develop step by step under the influence of the different forces.

[^13]
## THE THREE STRAIGHT-LINEAR FORCES:

## - The pushing forward and backward of the cosmic crosspath - SCORPIO:

In connection with the vertical advancing of the paths, the movements of Sun and Earth on the cosmic crosspath result in a zigzag path. Such a constant moving back and forth is inconceivable for such gigantic bodies as Earth and Sun are. Also under these circumstances a linkage of the sun to the outer planets, as Rudolf Steiner describes it, would not be possible. For that, the sun must come to rest. For this purpose, a force is needed which cancels the halfyearly forward and backward movements of the sun on the cross stem by a respectively exactly opposing pushing backward and forward of the cosmic crosspath. Taking into account the vertical advancing of the paths, this leads to the transformation of the sun's zigzag path into a vertical straight line due to an opposing zigzag force. Such a power we find very nicely symbolized in the consonant Z. Rudolf Steiner assigned it to the zodiac sign Scorpio (Eagle). Scorpio and Taurus face each other as a pair of opposites, as do the two formal principles of straight line and circle. The power of the zodiac sign Scorpio causes the apparent resting of the sun in the center of the earth's orbit as the basis of its linear movement in the direction of the solar apex. The force of the zodiac sign Taurus, on the other hand, causes the circular path of the sun through the zodiac.

## - The straight-linear forward movement (horizontal advancing) - SAGITTARIUS:

The linear solar path in the cosmos is not a vertical path, but it is inclined, in contrast to the earth's axis which is aligned to the north celestial pole. That part of the apex movement that takes place in the plane of the celestial equator corresponds to the horizontal advancing of the lemniscatory paths. The same process of movement can also be described as pulling the cosmic crosspath straight-linear in the distance. The six forces of the second half of the zodiac apparently have to do with the long-distance movement of the sun. By bringing the sun to rest, due to the zigzag force of Scorpio, the precondition of its linking to the outer planets and a distant movement in the cosmos is created. The newly added straight-linear horizontal movement of the sun (the horizontal advancing towards $0^{\circ}$ Capricorn) corresponds to the walking of man on earth. This movement is represented in the consonant $G$, which in the Greek gamma ( $\Gamma$ ) shows a composition of a vertical and a horizontal line. One can regard the vertical line as a mark of the starting point, from which the horizontal line, the actual horizontal G-movement (Go-movement), starts. Rudolf Steiner assigned this sound to the zodiac sign Sagittarius. Sagittarius is considered to be the sign of long-distance travel, both physical and spiritual (e.g. philosophical, religious).

## - The straight-linear backward inclination of the earth's axis - LIBRA:

To the extent that the earth and the sun are pulled horizontally into the distance by the force of the Sagittarius, the earth's axis is pulled further and further away from the north celestial pole at the same time. Inevitably there must be another force to counteract this effect. The further the earth moves in a straight line through the cosmos, the more its axis must lean back in a straight line, so that its connection with the north celestial pole can be maintained. Again, it is a cardinal sign that effects the axis regulation. The backward inclination of the earth's axis is symbolized to us in the Greek sound Chi, which is written like a Latin X. The two straight lines in X represent the play of inclination of an axis very clearly. The Greek sound Chi corresponds to the primal consonant "CH", which Rudolf Steiner assigned to the zodiac sign Libra. This is apparently the home of the third cosmic straight-linear force.

## THE THREE SUPERORDINATED BASIC FORCES:

## - Stability and Fixation of the Earth's axis - CAPRICORN:

Due to the effect of the three straight-linear forces, described above, on the cosmic crosspath the path of the sun is transformed into a straight-linear path (linear apex movement) and the path of the earth into a helical path around the straight path of the sun. In order to keep the Earth's axis aligned with the north celestial pole, although the earth is exposed to all the described, very diverse movements during its run on its lemniscatory path, it requires the three axis-regulating forces of cardinal signs Cancer, Aries and Libra. However, they can only do their work if they are given a common target point. The determination of this target point of the Earth's axis in the starry sky requires another, superordinated axial force. It apparently comes from the fourth cardinal sign, the stable and earthly sign Capricorn. By fixing the highest point of the Earth's axis in the starry sky, the degree of inclination of the Earth's orbit is determined with $23.5^{\circ}$ to the Earth's axis (see Figure 218, page 154) or from the lemniscatory view the degree of inclination of the Earth's lemniscate to the Sun's lemniscate. Thus the basic conditions for the celestial equator and the ecliptic as well as for the annual course of the sun through the zodiac are created. Rudolf Steiner has assigned the form-creating sound L to the zodiac sign Capricorn, which we use according to our linguistic spirit in words such as long (temporal) or length, longitudinal axis and location (spatial). The Greek letter lambda ( $\lambda$ ) can be interpreted as an image of this force, which supports the inclined earth's axis and thereby determines the degree of its inclination.

In addition, the force of stability has the effect that all form-shaping forces always act in exactly the same way according to fixed laws and thus a mathematically exactly calculable, rigid, stable system is maintained or would be maintained if there were not yet another counteracting force of instability or variability (see below). The effect of the power of Saturn's sign Capricorn of persisting and freezing makes the statement of Rudolf Steiner about Saturn's share in the path-forming process of the planets understandable: "If it only worked alone, we would only move in the lemniscate". (Lecture of 2 May 1920 [5]) The path-formation would be "frozen" at its initial stage, the self-contained lemniscate. A lemniscate that opens up and advances upwards as the next step of development would be prevented from the very beginning. Nevertheless, the power of Capricorn is of utmost importance. How else could a stable end result be achieved from the complicated path-forming process with so many different acting forces? The power of Capricorn has to regulate the stability in the planetary system. That this is also a moving process is expressed beautifully in the curved, slightly inclined handwritten form of the small Latin $\ell$.

This force may also be responsible for the fact that the earth moves closest to the Sun in January and binds itself more firmly to it, i.e. stabilizes its distance to the sun. Around January 2nd of each year, the sun reaches the perihelial distance. It then passes through $11^{\circ}$ Capricorn and thus towards the middle of the sign. If, on the other hand, the sun has its greatest distance to Capricorn and runs in the sign of Cancer, the earth also has its greatest distance to the sun. Around the 2nd of July of a year the sun passes the aphelial distance in about $10^{\circ}$ cancer. The earth's linking to the sun is then mostly relaxed. Thus, its linking to the sun through the stability force of the sign Capricorn would be consistent with the "speed solution" for the perihelion-aphel problem described in section 1.1.1.2.

## - Variability and Incomprehensibility - PISCES:

Now, the stabilizing force of Capricorn is on the other side opposed by a force of variability. Rudolf Steiner explains this fact with the words: "If we compare the orbital periods of the planets with each other, then incommensurable quantities emerge. For, if the sizes were commensurable, the planetary orbits would gradually come into such a relationship that the entire planetary system would become rigid. But in our planetary system there is this tendency to freeze, to become dead. If one takes the fact, which is given by expressing through certain curves and formulae what is present in the planetary system, and these curves and formulae, as we have seen, never agree entirely with reality, one has to say: If you try to grasp the appearances in the sky with easily transparent formulas or easily transparent figurative forms, the appearances slip out; they constantly slip away. Hence it is true: if we look out at the real picture of the appearances in the sky and then look at what we can do in the calculation, we will never get a formula that is completely consistent with the appearances. We can make a drawing like the one I designed for you yesterday as a system of lemniscates [see Figure 145, page 111, and Figure 224, page 167]; we can do that. But this system is only understood in the right way if one says: If I were to draw it concretely in any form, it could at best be the right thing for the present time. As soon as the time begins that I have given as the future ice age, I would have to modify this system in a substantial way, modify it so that I take the constants of the curve variable, and they themselves are quite complicated functions. Thus I can never draw simple lines, but I can only draw complicated lines. And even if I draw these lines here, I would actually have to say: Yes fine, I draw a path for any celestial body - we saw yesterday, it will always be a lemniscatory path. Yes, but after some time the necessity comes for me to no longer accept this drawing, but to make the lemniscate a little wider, and then I have to draw such a lemniscate and so on (Fig. 5 [see Figure 223]). That is, if I began to trace the paths of the celestial bodies, I would have to position myself in the universe and constantly follow the paths, constantly varying. I'm not even allowed to draw a constant path. I have to draw every path I trace with the awareness that I have to change continuously, because with every course of time I am challenged, that the path is a bit different again. Therefore, I'm not at all able to draw completed lines if I want to adequately conceive the celestial bodies with their paths. When I draw completed lines, they are approximated lines and I have to make corrections. This means that from every completed line afterwards slips out what is real in the sky. I may think of whatever a completed mathematical line, the real slips away from me, it doesn't fit in." (Lecture of 18 January 1921 [3])

Fig. 5


Figure 223: Rudolf Steiner's sketch on the variability of the planetary paths [3]

No other sign of the zodiac is so completely in harmony with the principles of variability, of slipping away and of not being fully comprehensible as the watery sign Pisces. If you see a fish clearly "standing" in front of you in the water and think you can grab it with your hands, it nonetheless quickly escapes, however courageous the grasp may be. The flowing, always variable and never fully comprehensible variability of the form creations of the water intensifies this impression of the incomprehensible. According to Rudolf Steiner, the formative force of the zodiac sign Pisces is expressed in the consonant N, the sound with which we negate a prescriptive: no, not (comprehensible), never ever. This force preserves the planetary system from freezing and falling out of the process of evolution.

## - Balance and Target tuning (Apex) - AQUARIUS:

After all, there must be a third superordinate force that balances all other forces so that ultimately those trajectories emerge on which the planets and the sun can make their way to the target. The entire system of trajectories must be tuned again and again to the destination of the journey of the planetary system in the cosmos, the solar apex. According to Rudolf Steiner, it is the planet Saturn, which leads all other planets in space or pulls behind itself. Thus, this target tuning power can only originate from another sign of Saturn. While the power of Capricorn tends to stiffness in its targeting and the power of Pisces to softening, the power of the sign Aquarius has the task of balancing, not only between the two powers of Capricorn and Pisces, but also between all the other forces of the zodiac involved. Only when everything is in perfect equilibrium and in agreement with the target point, can a continuous approach to the goal take place. In relation to the external form of man, the power of Aquarius brings about the balance between the circular and rotational forces of the head, the lemniscatory forces of the breast area and the straight-linear forces of the limbs. Once all three have been brought into the right balance, the human figure emerges as a microcosmic image of the Macrocosm. Therefore, the imaginative symbol of the sign Aquarius is the ideal figure of man, which is the prerequisite that he can pour the "waters of the Spirit" onto the earth. For the planetary system, the ideal figure is the external appearance of a helical path of the Earth around an inner straight-linear path of the Sun that has its target point in the solar apex. But such paths only come into being if a harmonious balance is created between all twelve formative forces. According to Rudolf Steiner, the formative force of Aquarius is expressed in the consonant M, which we use in words such as man, to harmonize, to comfort, to combine, to match etc.* In the handwritten, small Latin letter $\mathfrak{m}$ with its triple winding, the helical shape can still be understood.

The considerations on the formative processes of the planetary paths ultimately result in a selfcontained system of formative steps and formative forces. Therefore, instead of a lemniscatory path system we may speak as well of a system of formative-steps-system or formative-forcessystem. Those who only want to persist in the outer appearance can of course be content with the Copernican orbit of the earth around the sun and simply equate the shape of the orbit with the planetary impulse of movement. But as soon as modern astronomy sets off on its way to penetrate into the processes of path-formation, it will encounter a multitude of forces which, contradictory as they may be in part, nevertheless interact in perfect harmony and wisdom. Ultimately, these forces can only be imagined as living, intelligent force entities that, under the highest guidance, bring about the processes in the planetary system. For, behind the twelve planetary formative forces described above obviously there is a unified goal, a spiritual guidance that coordinates everything. Does not this remind us imminently of Christ, who, two thousand years ago, revealed Himself to humanity in bodily form and presented in the circle of the twelve apostles surrounding Him an image of his twelve-fold being?

[^14]In chapter III of the writing "The spiritual guidance of man and humanity" [18] Rudolf Steiner points out that materialistic science will be replaced by a Christ-permeated science, which will no longer cling to the idea that "there are only material atoms," but will recognize that "down to the smallest parts of the world, the substance is infused by the spirit of Christ. And strange as it may seem, in the future, chemists and physicists will come who teach chemistry and physics not as taught today ... but which will teach: matter is built up in the same sense as Christ gradually has arranged it! - One will find the Christ right down into the laws of chemistry and physics. A spiritual chemistry, a spiritual physics is what will come in the future. Nowadays, it certainly seems as a daydream or worse to many people today. But what is often the rationality of the times to come, that is folly for the preceding ones." The development towards a spiritual science certainly will not take place simultaneously in all branches of science. This raises the question: Which branch of science will precede? Where may we expect first insights in the direction of the announced spiritualization? - Rudolf Steiner answered this question clearly: "Astronomical science is the one that has the best opportunity of being led back into spirituality. With that, it's at the earliest possible to happen. " (Lecture of 7 January 1910 [9])

In the twelve primal consonants of the zodiacal forces Christ, the creative Cosmic Word, expresses Himself, through the twelve formative forces of the planetary paths, and in the midst of this circle of the Twelve He stands as the Thirteenth in the shape of the cosmic crosspath of Sun and Earth, His two most important domains as the Lamb of God (Figure 221, page 157, and Figure 222, page 160). In this sense, the present considerations may be a contribution to spiritualize astronomy as the first section of physics and to put it on a new foundation, a Christian foundation.

### 3.7 The lemniscatory paths of the outer planets

Finally, the task remains to examine the paths of the outer planets in order to determine whether they also follow the same laws as the described paths of the inner planets do. According to Rudolf Steiner, the task of the outer planets is to guide the sun and its associated planets Mercury, Venus and Earth on their long journey towards the solar apex in the constellation of Hercules. In doing so, they protectively cover the inner solar system or take it in their midst. Rudolf Steiner described this process with the words: "... an outer planet takes up the Earth-Sun path in its loop." However, it does not remain immobile within one half of the lemniscate, but seems to swing back and forth between the two halves, because he goes on to say: "But now the lemniscate proceeds, so it pushes itself through through this lemniscate, which represents the outer planets".* (Lecture of 17 January 1921 [3], Fig. 7, here as Figure 224) The latter lemniscate in Rudolf Steiner's sketch is apparently supposed to represent the lemniscate of Mars. Consequently, Mars must carry the entire inner solar system with it while running on its own lemniscate. A stable linking between Sun and Mars is only possible if the sun "rests", so that it can follow the guidance of Mars faithfully. How such a resting sun is created in the lemniscatory system has already been described above in the section "The moving cosmic crosspath and the resting sun".

The actual draught horse of our planetary system is Saturn: "... if we take this outermost planet of our solar system, Saturn, then we have to imagine it ... as the leader of our planetary system in world space. It pulls our planetary system in the world space. It is the body of the outermost force that guides us around in the lemniscate in the world space. It drives and pulls at the same time. So it is the force of the outermost periphery." (Lecture of 2 May, 1920 [5]) In another lecture Rudolf

[^15]Steiner adds: "So, it is not that the planets move around the sun, but these three: Mercury, Venus, Earth, follow the sun, and these three: Mars, Jupiter, Saturn go ahead." (GA 300 a, conference of 25 September 1919). This statement was illustrated in the same lecture by the sketch of a helical line with the note: "The helical line continues in space" (Figure 225). However, this figure cannot be understood as if all the planets were running one after the other being just lined up like a string of pearls. We can observe from the earth how the planets move away from each other at times so far that they can even come in opposition to each other. If one wanted to depict a Saturn-Jupiter opposition in the sense of the sketch given with Saturn remaining at the left edge of the picture, Jupiter would have to be drawn below the earth and outside of the right edge of the figure. For, the earth would then stand on a line between Saturn and Jupiter. The curved arrow coming from the earth indicates such additional movements of the individual planets. So once again the sketch only shows a certain relation of forces in principle.


Figure 224: Rudolf Steiner's sketch on the lemniscatory paths of the inner and outer planets [3]


Figure 225: Rudolf Steiner's sketch on the helical path of the planets [4]

How the idea of lemniscatory paths of the outer planets can be reconciled with their Copernican orbits is the subject of the following observations.

### 3.7.1 The Lemniscate of Mars

In Rudolf Steiner's sketch of the lemniscatory paths of the outer planets, only a single lemniscatory path is depicted as a representative of them (see above, Figure 224). Judging by the size ratios, it seems to be the lemniscate of Mars. The sun stands in the middle of a lemniscate half of the path of Mars. As with the inner planets, the question arises here: "Does the Copernican orbit of Mars correspond to a lemniscate half of its path?"

In Figure 226, the Copernican Mars orbit was entered in Rudolf Steiner's sketch, taking into account the distance between Earth and Sun given therein. It shows that the Copernican Mars orbit is a good deal larger than the sketched lemniscate half of Mars. Certainly, Rudolf Steiner did not want to draw to scale here, but simply depict the connections in principle. In any case, he considered at least the Earth-Sun lemniscate - and thus also the cosmic crosspath - in the sketch not as resting, but as in motion, because he said: "But now the lemniscate proceeds, so it pushes itself through, through this lemniscate, which represents the outer planets". (Lecture from 17 January 1921 [3]) How should this be possible? How should the Earth-Sun lemniscate get across to the other lemniscate half of Mars or, since Mars passes through both lemniscates halves one after another, oscillate between the two? After all, the sun must keep its distance to Mars.


Figure 226: Copernican Mars orbit entered in Rudolf Steiner's sketch

The first step to a solution of the question, as with the considerations of the lemniscatory paths of the inner planets, is first of all to clarify the orientation of the Mars orbit in space or in the zodiac (Figure 227). Although the Copernican Mars orbit with 0.0935 has less eccentricity than the Mercury path, whose eccentricity is 0.2056 , it is still shaped elliptical. In the figure, therefore, the sun is not exactly in the middle of the Mars orbit, but it is clearly offset with the surrounding Copernican orbits of the inner planets to the upper left. If you draw a straight line from the point on the Mars orbit furthest from the sun (Aphelion) to the point closest to the sun (Perihelion), the resulting line shows in 2011, seen from the sun, towards about $6^{\circ}$ Pisces.

The observations of the lemniscate tracts of the inner planets revealed that the perihelion of the Copernican orbit is identical to the lemniscate midpoint. Following this insight, Figure 228 shows the position of the lemniscate of Mars at that time. Mars passed through its lemniscate midpoint on 21 April 2009. Seen from the earth, this was in the direction of $29^{\circ}$ Pisces. Mars requires 687 days $=1.88$ years for a complete round on its elliptical orbit. This period corresponds to its "Copernican year". For a full round on the marked lemniscatory path, a "lemniscatory year", it needs twice the time of 1,374 days or 3.76 years. A "lemniscatory month" for Mars corresponds to one twelfth of this: 114.5 days (almost four solar months).


Figure 227: Orientation of the Mars orbit in the zodiac

Figures 228 through 234 show the trajectory of Mars for six lemniscatory months, if you allow its lemniscate to rotate like the paths of the inner planets, i.e. counterclockwise. The rotation angle is on average $-60^{\circ}$. The elliptical shape of the Mars orbit requires a variability similar to the Mercury orbit. While the rotation angle of Mercury deviates by $\pm 10^{\circ}$ from the mean value, here it is only $\pm 3^{\circ}$. The exact angles of rotation are initially descending $-63^{\circ},-60^{\circ},-57^{\circ}$ and then ascending again $-57^{\circ},-60^{\circ},-63^{\circ}$. In total, a full $360^{\circ}$ rotation of the lemniscate takes place in half a lemniscatory year of 687 days (a Copernican Martian year). Mars is running clockwise on its lemniscate path and yet seems to run counterclockwise on an elliptical orbit simultaneously (see Figure 229). The elliptical orbit is the outward appearance that results from the interaction of planetary lemniscatory movement and path rotation. If, for closer inspection, the Martian motion is checked at intervals of half a lemniscatory month of 57 to 58 days, the rotation angle for the lemniscate is $-33^{\circ}$ immediately after the perihelion passage, then four times $-30^{\circ}$ and still $-27^{\circ}$ to the aphelion position. The result is: $\left(-33^{\circ}\right)+\left(-30^{\circ}\right)=-63^{\circ}$ and $\left(-30^{\circ}\right)+\left(-30^{\circ}\right)=-60^{\circ}$ and $\left(-30^{\circ}\right)$ $+\left(-27^{\circ}\right)=-57^{\circ}$ and then the reverse, ascending order $-57^{\circ},-60^{\circ}$ and $-63^{\circ}$.

On 9 March 2011 Mars reaches again its perihelion in the lemniscate midpoint (Figure 234). In the sense of the power of Gemini described above, which causes a half-yearly rotation followed by half-yearly rest on the lemniscatory paths of Mercury, Venus, Earth and Sun, the lemniscate of Mars also rests during the second half of its lemniscatory year. Mars is now running counterclockwise, following the change of direction in the lemniscate. In doing so, the lemniscatory path completely covers the Copernican elliptical orbit. Only after the passing of 1.88 years, when Mars has regained its perihelion in the lemniscate midpoint, does the lemniscate of Mars perform a full rotation in half a year. It can be clearly seen from Figures 229 to 231 how, as a result of the rotation of Mars' lemniscate, the earth's orbit with the sun gradually get over from one half of the lemniscate (thin red circle) to the other half of the lemniscate (thick red circle) and then to Figs. 232 to 234 returns to the original lemniscate half.


Figure 228: Position of the Mars lemniscate on 21 April 2009


Figure 229: Position of the Mars lemniscate on 13 August 2009


Figure 230: Position of the Mars lemniscate on 6 December 2009


Figure 231: Position of the Mars lemniscate on 31 March 2010


Figure 232: Position of the Mars lemniscate on 24 July 2010


Figure 233: Position of the Mars lemniscate on 15 November 2010


Figure 234: Position of the Mars lemniscate on 9 March 2011


Figure 235: Proportions of the Mars lemniscate and the lemniscatory paths of the inner planetary system as well as the Copernican orbits

The proportions between the lemniscate of Mars and the lemniscatory paths of the inner planetary system are illustrated in Figure 235. The Copernican orbits of Earth and Venus are shown in dashed lines. The Copernican orbit of Mercury in the figure is identical to its upper lemniscate half as is the case at its perihelion passage (see Figure 150, page 116). The position of the Mars lemniscate is also marked for the time of the perihelion passage of Mars (cf. Figure 234). The Mars lemniscate extends far beyond the Copernican orbit in the direction of the Jupiter orbit, but it does not intersect it, as is the case with the Venus lemniscate with regard to the Mars lemniscate. The endpoints of the two lemniscate halves of the Mars orbit (aphelion points) can be up to 4.428 AU distance from the sun. This is the sum of the perihelial distance Mars-Sun (1.381 AU) plus the longitudinal diameter of its lemniscatory half (= perihelial distance $1.381+$ aphelial distance 1.666 AU ). This means that Mars does not reach the closest point of the Jupiter orbit to the sun. This lies at a distance of 4.95 AU from the sun.

### 3.7.2 The Lemniscate of Jupiter

The Copernican Jupiter orbit is so gigantic that it includes the Mars lemniscate and the entire inner planetary system. But, this orbit is only one half of the Jupiter's lemniscatory path. The actual path of Jupiter is thus much larger than the Copernican orbit that arises when one summarizes the externally observable Jupiter positions in an elliptic orbit. The perihelion of the Jupiter orbit points in the cosmos towards $15^{\circ}$ Aries (Figure 236). It is again the midpoint of the lemniscatory path. Jupiter ran on 28 March 2011 through its perihelion. From Earth's perspective, it was then standing in $14^{\circ}$ Aries (Figure 237). To compare sizes, the Copernican Saturn orbit is shown in addition to the Copernican orbits of Mars and Earth. The Jupiter lemniscate extends far beyond the Copernican Saturn orbit, much further than the Venus lemniscate reaches beyond the Copernican Mars orbit, as shown by a comparison with Figure 235.

Jupiter needs 11.86 years to complete a circuit on his Copernican orbit. A full round on his lemniscatory path lasts twice as long: 23.72 years. This corresponds to a lemniscatory year of Jupiter. A lemniscatory month of Jupiter thus lasts 722 days, i.e. a little less than 2 years. The following illustrations show Jupiter's movements in intervals of six lemniscatory months. The lemniscatory path rotates on average by $-60^{\circ}$ per lemniscatory month. Due to the eccentricity of Jupiter's orbit, this value varies by $\pm 3^{\circ}$. The angles of rotation are $-63^{\circ},-60^{\circ},-57^{\circ}$ and then in reverse order $-57^{\circ},-60^{\circ}$ and $-63^{\circ}$. Jupiter is running clockwise on the rotating lemniscatory path. At the same time it seems to run counterclockwise on an elliptical orbit. Although it cannot be said that Jupiter revolves around the Sun because of the lemniscatory mode of movement, its positions, when joined together, give exactly that elliptical orbit described by Copernican astronomy.

On 2 March 2017 Jupiter will pass through the aphelion of its Copernican orbit. It then reaches one of the two path positions which are furthest away from its lemniscate midpoint. A quarter of a lemniscatory year later, on 7 February 2023, it will again run through its perihelion or the lemniscate midpoint. Then the Jupiter lemniscate rests for half a lemniscate year or a Copernican Jupiter year, as it was also described for the lemniscatory paths of the other planets. During this time, the path to be travelled by Jupiter completely coincides with the Copernican orbit, giving the impression even more clearly of Jupiter moving around the Sun in this period.

Figures 238 to 240 show how, by rotation of the Jupiter's lemniscate, the earth's orbit with the sun gradually gets from one half of the lemniscate (thin orange circle line) to the other half of the lemniscate (thick orange circle line) and then returns to the original half of the lemniscate in figures 241 to 243.


Figure 236: Orientation of the Jupiter orbit in the zodiac


Figure 237: Position of the Jupiter lemniscate on 28 March 2011


Figure 238: Position of the Jupiter lemniscate on 19 March 2013


Figure 239: Position of the Jupiter lemniscate on 11 March 2015


Figure 240: Position of the Jupiter lemniscate on 2 March 2017


Figure 241: Position of the Jupiter lemniscate on 24 February 2019


Figure 242: Position of the Jupiter lemniscate on 15 February 2021


Figure 243: Position of the Jupiter lemniscate on 7 February 2023

Figure 244 gives an overview of the size ratios between the Jupiter lemniscate and the smaller lemniscatory paths encompassed by it. The Copernican orbits of Earth and Venus are shown in dashed lines. Mercury's Copernican orbit is identical to its upper lemniscate half as it is the case in its perihelial passage. The position of the Mars lemniscate is also indicated for the time of the perihelion passage. All paths fit completely into a lemniscate half of the Jupiter path. However, they are not always located within such a half as the Jupiter lemniscate rotates, as described above.


Figure 244: Proportions of the Jupiter lemniscate and the lemniscates encompassed by it

### 3.7.3 The Lemniscate of Saturn

Even more gigantic than the already extremely large lemniscate of Jupiter is the lemniscate of Saturn. The Copernican path of Saturn corresponds again only to one half of its lemniscatory path. The perihelion of the Saturn orbit is at $2.6^{\circ}$ Cancer, seen from the sun (Figure 245). It forms the midpoint of the lemniscatory path, around which the path rotates. On 26 July 2003, Saturn passed through the perihelion. Viewed from Earth, it was in $7^{\circ}$ Cancer. In Figure 246, also the Copernican orbits of Jupiter, Mars and Earth are shown for size comparison.

Saturn needs 29.457 years to complete a circuit on its Copernican orbit. A full round on its lemniscatory path lasts twice as long: 58.914 years. This corresponds to a lemniscatory year of Saturn. A lemniscatory month of Saturn therefore takes 4.91 years. That's 33 days less than 5 years. The following figures show the movements of Saturn in intervals of six lemniscatory months. The lemniscatory path rotates on average - $60^{\circ}$ per lemniscatory month. Since the eccentricity of the Saturn orbit differs only slightly from that of the Jupiter orbit, the value fluctuates also here by $\pm 3^{\circ}$. The angles of rotation are again $-63^{\circ},-60^{\circ},-57^{\circ}$ and then in reverse order, but in the same direction $-57^{\circ},-60^{\circ}$ and $-63^{\circ}$.


Figure 245: Orientation of the Saturn orbit in the zodiac


Figure 246: Position of the Saturn lemniscate on 26 July 2003


Figure 247: Position of the Saturn lemniscate on 21 June 2008


Figure 248: Position of the Saturn lemniscate on 18 May 2013


Figure 249: Position of the Saturn lemniscate on 14 April 2018


Figure 250: Position of the Saturn lemniscate on 11 March 2023


Figure 251: Position of the Saturn lemniscate on 6 February 2028


Figure 252: Position of the Saturn lemniscate on 3 January 2033

On the counterclockwise rotating lemniscate, Saturn runs clockwise for half a lemniscatory year. At the same time, however, it seems to run counterclockwise on an elliptical orbit. Although, due to the lemniscatory course of motion, it cannot be said that Saturn revolves around the Sun, its positions, when joined together, yield exactly that elliptical orbit described by Copernican astronomy.

On 14 April 2018 Saturn will go through the aphelion of its Copernican orbit. It then reaches one of the two positions furthest away from its lemniscate midpoint (Figure 249). A quarter of lemniscatory year later, on 3 January 2033, it will again pass through its perihelion or lemniscate midpoint (Figure 252). From then on, the Saturn lemniscate will rest for half a lemniscatory year or a whole Copernican Saturn year, according to the regularities also described for the other lemniscatory paths. During this period, the path to be travelled by Saturn completely coincides with the Copernican orbit. Finally, Figure 253 provides an overview of the size relationships between the Saturn lemniscate and the smaller lemniscatory paths encompassed by it.


Figure 253: Proportions of the Saturn lemniscate and the lemniscates encompassed by it

## Epilogue

In the three-part series of considerations "The Lemniscatory Path System" completely new motion sequences of the planets were described. Nevertheless, they are not in contrast to the Copernican observations. The lemniscatory path system also results for all planets ultimately in exactly those elliptical orbits that modern astronomy describes. However, they are created in a very different way, namely through the interaction of twelve formative forces of the planetary orbits. The pathforming process leads through five steps, with a cosmic crosspath of Sun and Earth with their straight-linear paths in the center. Understandably, such complicated processes will, in the foreseeable future, meet with little approval from astronomers. But did not the heliocentric system of Copernicus go exactly the same way after the first publication? The new and necessary takes a long time until it gains acceptance. In the beginning it is usually not recognized in its importance and, as it differs so much from the fixed ways of thinking, it is often rejected as being not up-todate. Rudolf Steiner described this problem, which also arose during the transition from the Ptolemaic to the Copernican system, with the words: "Up-to-date was the impetus for a new view of space that Copernicus gave in the dawn of the new era, undoubtedly up-to-date from the point of view that the development of mankind made it necessary that this impulse came, especially at the time of Copernicus. Not up-to-date this impulse proved to be for a long time, inasmuch as it was opposed by all those who wanted to adhere to the old ways of thinking, to centuries-old and millennia-old ideas." (Lecture of 6 April 1914 [16]) Similarly, the further development of the Copernican system towards a lemniscatory path system or system of formative forces, as it was developed here, based on Rudolf Steiner's specifications, will also take place. But as great as the resistances will be, ultimately humanity will have to take the step towards understanding the cosmic formative forces of the planetary paths in order to ascend from the realm of the dead form to the realm of the moving life, the ethereal, out of which the form observable with physical senses is first born. In this sense, some statements by Rudolf Steiner are repeated in which he points out the necessity for today's humanity to rise from pure formal thinking to a thinking in movements:
"The whole is in movement, and every single thing that comes out by the movement is a selfcontained form. ... Hence we see: the philosophers of Nominalism, who necessarily stand at a boundary-line, move within a certain realm, the realm of the Spirits of Form. Within this realm of the Spirits of Form, which is all around us, forms are prevalent; and since the forms are prevalent, in this realm we find single, strictly self-contained separate things. From this you can see that the philosophers, whom I mean, have never made the decision to leave the realm of forms ... If they would leave the realm of special things, i.e. forms, they would enter into an imagination that is in continuous movement, i.e. they would have in their thinking a realization of the realm of the Spirits of Movement, the next higher hierarchy." (Lecture of 20 January 1914 [15])

Elsewhere, Rudolf Steiner explains: "Astronomical science is the one that has the best opportunity of being led back into spirituality. With that, it's at the earliest possible to happen." (Lecture of 7 January 1910 [9]) How this spiritualization will look like is described in chapter III of his book "The Spiritual Guidance of Man and Humanity" [18]. There Rudolf Steiner speaks of the replacement of materialistic science by a Christ-permeated science, which will no longer cling to the idea that "there are only material atoms," but will recognize that "down to the smallest parts of the world, the substance is infused by the spirit of Christ. And strange as it may seem, in the future, chemists and physicists will come who teach chemistry and physics not as taught today ... but which will teach: matter is built up in the same sense as Christ gradually has arranged it! One will find the Christ right down into the laws of chemistry and physics. A spiritual chemistry, a spiritual physics is what will come in the future. Nowadays, it certainly seems as a daydream or worse to many people today. But what is often the rationality of the times to come, that is folly for the preceding ones."

In the twelve primal consonants of the zodiacal forces the Christ, the creative Cosmic Word, expresses Himself through the twelve formative forces of the planetary paths, and in the midst of this circle of the Twelve, He stands as the Thirteenth in the form of the cosmic cross orbit of the sun and the earth, the two "planets" which are His main ministries as the Lamb of God (compare Figure 221, page 157, and Figure 222, page 160). In this sense, the present considerations may be a contribution to spiritualize astronomy as the first part of physics and to put it on a new, a Christian basis.

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GRATIA JOVIS SPHAERAE CHERUBINIBUS
SEPTENNII OCTAVI REGNATORIBUS
MINISTRISQUE SUIS
$\sum$


[^0]:    * Such was the situation in 2012. After release of the complete work on the Internet for free download in autumn 2012, an additional publication of PART 3 in JUPITER two years later had become superfluous.

[^1]:    * At least not as a final, fully understood reality, but only as part of a higher, more comprehensive reality.
    ** By the ,,geometric external image" Rudolf Steiner apparently meant the physically observable end result (the Copernican planetary orbits), which results from processes which he summarized under the term „the other image" (the path-forming processes based on lemniscatory paths.)

[^2]:    * See Figure 9 (Page 12). If Rudolf Steiner's sketch shown there is turned $90^{\circ}$ to the advancing lemniscate and mentally pulled apart horizontally, a curve progression is obtained, which basically corresponds to the curve progression in Figure 2.

[^3]:    * $\mathrm{AU}=$ astronomical unit $=$ mean distance between Earth and Sun, about 149.6 million km or 93 million miles)

[^4]:    * As the Sun and Earth move upwards on their lemniscatory paths, the vertical was given as the apex direction. This is only a first working model in order to be able to get into the consequences of such paths at all. Only in Part 2 the problem of the alignment of earth axis, sun axis, lemniscate axis and apex direction in the Lemniscatory Path System will be discussed in detail.

[^5]:    * (that looping occurs because a planet moving closer to the sun overtakes another planet running further around the sun)

[^6]:    * Maybe Rudolf Steiner actually used the word "through" twice in the sense of: expresses itself through, through this lemniscate.

[^7]:    * The 5th cultural period extends to Rudolf Steiner from 1413 to 3573.
    ** The centers of the orbits of the planets are not all in the same point within the Sun (center), but are different from it (eccentric).

[^8]:    See Figure 54 (page 36)
    ** GA 151 "Human and cosmic thought"

[^9]:    Compare with Figure 87 (page 58)

[^10]:    * The original Latin text by Copernicus in the first edition of "De Revolutionibus orbium coelestium" from the year 1543 reads: "In medio vero omnium residet Sol. Quis enim in hoc pulcherrimo templo lampadem hanc in alio vel meliori loco poneret, quam unde totum simul possit illuminare? Siquidem non inepte quidam lucernam mundi, alii mentem, alii rectorem vocant. Trimegistus visibilem Deum, Sophoclis Electra intuentem omnia. Ita profecto tanquam in solio regali Sol residens circum agentem gubernat Astrorum familiam. Tellus quoque minime fraudatur lunari ministerio, sed ut Aristoteles de animalibus ait, maximam Luna cum terra cognationem habet. Concipit interea a Sole terra, et impregnatur annuo partu." Liber I, Cap. X, pagina 9b)

[^11]:    * Copernicus was baptized on the name of his father: Niklas Koppernigk. The prince-bishopric Warmia (Ermland) where he lived was a Prussian settlement area, subordinated to Pope in Rome, which kept its German culture and German language throughout the lifetime of Copernicus and long after that, although, on the other hand, it has been increasingly integrated into the territory of the Polish king.
    ** German for "Our Lady's fortress". Today Polish: Frombork

[^12]:    * Today Polish: Lidzbark Warmiński

[^13]:    * The German original text uses the word "Rad" here (English: wheel).
    ** The English term "double U" expresses this very nicely. However, each consonant of a sign always appears in two variants, as voiced and voiceless, hard and soft (e. g. P and B, T and D, K and G etc.) The voiceless variant of "W" is "V", as it appears in the word "vault". In the German language the "W" is spoken like the English "V".

[^14]:    The German original text has the words "Mensch" (man), "abstimmen" (to tune), "abschmecken" (to taste), "anschmiegen" (to cuddle), "zusammenstimmen" (to harmonize, to match).

[^15]:    * It is possible that Rudolf Steiner actually used the word "through" twice in the sense of: "so it pushes itself through, through this lemniscate."

